KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE MATHEMATICS /BACHELOR OF EDUCATION SCIENCE/ARTS

SECOND SEMESTER 2022/2023

[JUNE - SEPTEMBER, 2022]

MATH 210: LINEAR ALGEBRA I

STREAM: Y2 S2 **TIME: 2 HOURS**

DAY: MONDAY, 9.00 AM – 11.00 AM **DATE: 05/09/2022**

INSTRUCTIONS:

1. Do not write anything on this question paper.
2. Answer question ONE [Compulsory] and any other TWO Questions.
3. Show ALL necessary working.

Question One (30marks)

1. a)i) State two conditions for set of a Vectors V to be a vector space over a field F of scalar k (2marks)

ii) Show that the Set $V = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}$ is a vector space over the field of scalars K (5marks)

b)i) for any vector space V and $V_{1,}V_2 \dots \dots V_n \in V$ state two the condition for V to be a liner combination of $V_{1,}V_2 \dots \dots V_n$ (2marks)

ii) Show that any vector $V = (a, b)$ is a linear combination of $V_1 = (1,0) and\ V_2 = (0,1)$ ``` (4marks)

c)i) State the condition for a set of vectors $V_{1,}V_2 \dots \dots V_n$ to be linearly independent (2marks)

ii) Hence show that the set of vectors $u = (0,5,-3,1), v = (6,2,3,4) and\ w = (0,0,7,-2)$ are linearly independent (7marks)

d) i) Define a basis for $V = \mathbb{R}^n$ (2marks)

ii) State the basis for each of the following vector spaces (4marks)

$V = \mathbb{R}^3$ and V = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

e) For a transformation mapping $T$and sets of vector $v\ and\ u$, state the condition for $T: V \rightarrow U$to be a linear transformation (2marks)

QUESTION TWO (20MARKS)

2. a) i) Given a vector space V, state three conditions for a set of vector W to be a subspace of $V(W \underline{c} V)$ (3marks)

iiLet $V = \mathbb{R}^3 = (x, y)$ and$w = \left\{(x, y): x + y + 1 = 1\right\}$show that$W \underline{c} V$, with respect to the three condition in (a)(i) above (7marks)

b) Show that the vectors $V_1 = (1, -1, 0), V_2 = (1, 3, -1)$ and $V_3 = (5, 3, -2), V_2$ are linearly dependent (10marks)

QUESTION THREE (20MARKS)

3. a) i) What is the condition for a set of vector $V_{1,} V_2 \ldots \ldots V_3$ to be a linear span for a vector V (2marks)

ii) Show that the set $V_1 = (1,2,3), V_2 = (0,1,2)$ and $V_3 = (0,0,1),$ is a linear span for $V = R^3 = (a, b, c)$ (7marks)

iii) Confirm your result in (a) (ii) above for $v \in V = (2, -5, 1)$ (3marks)

b) Given$W = \left\{(x, y, z, w) \in R^4 : x + y = z - w = 0\right\}$find the basis for$W$ and state the dimension for$W$ (8marks)

QUESTION FOUR (20MARKS)

4. a) Show that $V = (7,10,16)$ is a linear combination of the vectors $a = (2,0,2), \mathrm{b} = (1,2,0) and\ c = (0,1,3)$ (10marks)

b)Show that $\mathrm{T(x, y) = (x - y, 2y)}$ is a linear transformation (1omarks)

QUESTION FIVE (20MARKS)

5. a) Let$T(x, y, z) = (2x + y, x - y)$ be a linear transformation. Find the matrix A of the linear transformation (10marks)

b)Using Gauss Jordan Method, solve the system of linear equations (10marks)

$$x_1 + 6x_2 + 4x_5 = -2$$
$$x_3 + 3x_5 = 1$$
$$5x_4 + 2x_5 = 2$$