KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE [MATHEMATICS]

SECOND SEMESTER 2022/2023

[JUNE - SEPTEMBER, 2022]

MATH 318: CALCULUS III

STREAM: Y2 S2 **TIME: 2 HOURS**

DAY: TUESDAY, 12.00 PM – 2.00 PM **DATE: 13/09/2022**

INSTRUCTIONS:

1. Do not write anything on this question paper.

2. Answer question ONE [Compulsory] and any other TWO Questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Evaluate $\int_0^3 \int_0^x \int_0^{x-y} 4xy \, dzdydx$(3marks)

b) Use Alternating series Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$ (2marks)

c) Test for the convergence of the sequence$\frac{3}{4}, \frac{6}{5}, \frac{9}{6}, \frac{12}{7}, \frac{15}{8}, \ldots$.. (4marks)

d) Apply the Integral Test to the series $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$ to determine whether it converges or diverges. (4marks)

e) A ball is dropped from a height of 8 metres and begins bouncing. The height of each bounce is three-fourths the height of the previous bounce. Find the total vertical distance travelled by the ball. (4marks)

f) Determine whether the series $\sum_{n=2}^{\infty}\frac{1}{nlnn}$ converges or diverges by Integral Test. (4marks)

g) If $u = f(x, y) = 3x^4 + 5x^2y - 7y^3$, find:

i)$\frac{\partial^2 u}{\partial x^2}$ ii) $\frac{\partial^2 u}{\partial y^2}$ iii) $\frac{\partial^2 u}{\partial x \partial y}$ (5marks)

h) Determine whether the series $\sum_{n=1}^{\infty}\frac{n^3}{n^7+9}$ converges or diverges using Limit Comparison Test.

(4marks)

QUESTION TWO (20MARKS)

a) Find the optimum of $f(x,y) = 8x^2 + 20y^2$ given that $x^2 + y^2 \leq 4$ using Lagrange Multipliers. (6marks)

b) Determine whether the series $\sum_{n=0}^{\infty} \frac{(-1)^n 5^{n+2}}{(n+1)^n}$ converges by Root Test. (4marks)

c) Find the extremals of the functional $\int_{x_1}^{x_2} \left(\frac{y'^2}{x^3}\right) dx$ (4marks)

d) Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ by Ratio Test. (6marks)

QUESTION THREE (20MARKS)

a) Using Lagrange method, find the greatest and smallest values that the function $f(x,y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$. (5marks)

b) Test for an extremum of the functional $I[y(x)] = \int_0^1 (xy + y^2 - 2y^2 y')\, dx$, $y(0) = 1$, $y(1) = 2$.
(4marks)

c) Using Direct Comparison Test determine the convergence of $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+2}$ (3marks)

d) Find the maximum or minimum values of the function $f(x,y) = 8x^2 + 12xy + 18y^2 - 16x - 48y - 8$. (8marks)

QUESTION FOUR (20MARKS)

a) Using the method of differences test determine the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^2+7n+12}$ (5marks)

b) Determine if the infinite series converges or diverges by using the Direct Comparison Test. $\sum_{n=1}^{\infty} \frac{cos^2 n}{n^2+3}$ (3marks)

c) Show that $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ is an improper integral and investigate its convergence. (6marks)

d) Evaluate $\iint_R (2x^2 + y)\, dxdy$ where R is the region bounded by the lines $y = x$ and the curve $y = x^2$. (6marks)

QUESTION FIVE (20MARKS)

a) Use integral test to determine convergence of $\sum_{n=1}^{\infty} \frac{4n}{6n^2+8}$ (8marks)

b) Investigate the convergence of the improper integral $\int_0^{\infty} \frac{2x dx}{(x^2+1)^2}$ (6marks)

c) Show that the integral is improper and evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ (6marks