

KISII UNIVERSITY
SCHOOL PURE AND APPLIED SCIENCES
DEPARTMENT MATHEMATICS AND
ACTUARIAL SCIENCE
COURSE TITLE:INTRODUCTION TO
MATHEMATICS FOR ACTUARIAL
SCIENCE
COURSE CODE: BACS 110
FINAL EXAM JUNE 2022
INSTRUCTIONS:Answer question one and
any other 2 questions in section B

SECTION A (30 marks)

Question One (30 marks)

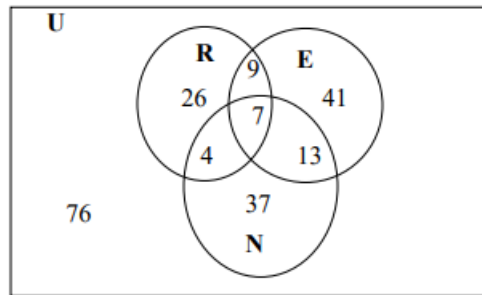
- (a) Prove the following statement. (5mks)

$$[(p \rightarrow r) \vee (q \rightarrow s)] \equiv [(p \wedge q) \rightarrow (r \vee s)].$$

- (b) Determine if the following statements are true or false. (5mks)

- (i) If triangles are polygons, then circles are ellipses.
- (ii) If $2 \times 4 = 8$, then $2 + 4 = 8$.
- (iii) Horses are mammals and frogs are amphibians.
- (iv) All birds cannot fly or all mammals can swim.

- (v) Mark Twain was a famous athlete or actor.
- (c) Check the validity of the following argument: If it is raining, the picnic is canceled. If it is not raining, the soccer game is not delayed. But the picnic is not canceled. Therefore, the soccer game is not delayed. (5mks)
- (d) Use the diagram below to find the cardinality for each problem. (5mks)



a. $n(N) =$ _____ b. $n[(R \cap E) \cup N] =$ _____ c. $n(E') =$ _____

- (e) Use Cramer's rule to solve for x, y and z . (5mks)

$$\begin{aligned} 2X + 3Y - Z &= 1 \\ 4X + Y - 3Z &= 11 \\ 3X - 2Y + 5Z &= 21 \end{aligned}$$

- (f) Assume that $A = \{1, 2, 3, \dots, 14\}$. Define a relation R from A to A by $R = \{(x, y) : 3x^2y = 0, \text{ such that } x, y \in A\}$. Determine and write down its range, domain, and codomain. (5mks)

SECTION B

Question Two (20 marks)

- (a) Find truth table of the following statement. (5mks)

$$[(p \rightarrow r) \wedge (q \rightarrow r)] \Leftrightarrow [(p \vee q) \rightarrow r]$$

- (b) If an object is less dense than water, it will float. The water displaced by an object will weigh more than the object, or the object will not float. But the water displaced by the object does not weigh more than the object. Thus, the object is not less dense than water. Prove validity of this statement. (5mks)
- (c) If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$. Find $A \cup B \cup C$. (2mks)
- (d) If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$. Find $A \cap (B \cup C)$ (3mks)
- (e) Evaluate the following:
- There are 8 men and 10 women and you need to form a committee of 5 men and 6 women. In how many ways can the committee be formed? (2mks)
 - How many 3 letter words with or without meaning can come out of letters of the word LOGARITHMS, if repetition of letters is not allowed? (3mks)

Question Three (20 marks)

- (a) Find the truth value of the following statement. (5mks)

$$\left[p \vee (q \wedge r) \right] \vee \sim q$$

- (b) Proof the validity of the following argument. If the Empire State Building is tall, then it will be struck by lightning. If the Empire State Building is not tall, it will not touch the clouds. The Empire State Building is not struck by lightning. Therefore, the Empire State Building is not tall (5mks)
- (c) In a class of 120 students numbered 1 to 120, all even numbered students opt for Physics, those whose numbers are divisible by 5 opt for Chemistry and those whose numbers are divisible by 7 opt for Math. How many opt for none of the three subjects? (3mks)
- (d) In a class of 40 students, 12 enrolled for both English and German. 22 enrolled for German. If the students of the class enrolled for at least one of the two subjects, then how many students enrolled for only English and not German? (2mks)
- (e) Find the inverse of the following function $f(x) = 4x^3 + 2$ (3mks)
- (f) Consider the relation "is a factor of" from the set $A = \{2, 3, 4, 5, 6\}$ to the set $B = \{6, 8, 10, 12\}$ Make an arrow diagram of this relation. (2mks)

Question Four (20 marks)

- (a) Find the truth values for the following statement. (5mks)

$$\left[(p \rightarrow q) \wedge (q \rightarrow r) \right] \vee (p \rightarrow r)$$

- (b) Proof validity of this statement. (5mks)

$$\begin{array}{l} 1 \quad (B \vee N) \Rightarrow (K \wedge L) \\ 2 \quad \neg K \\ 3 \quad \neg M \quad / \therefore \neg B \wedge \neg M \end{array}$$

- (c) Of the 200 candidates who were interviewed for a position at a call center, 100 had a two-wheeler, 70 had a credit card and 140 had a mobile phone. 40 of them had both, a two-wheeler and a credit card, 30 had both, a credit card and a mobile phone and 60 had both, a two wheeler and mobile phone and 10 had all three. How many candidates had none of the three? (3mks)
- (d) In a class 40% of the students enrolled for Math and 70% enrolled for Economics. If 15% of the students enrolled for both Math and Economics, what % of the students of the class did not enroll for either of the two subjects? (2mks)
- (e) What is the n th term of the sequence 5, 7, 9, 11, ... (2mks)
- (f) What is the 50th term of the sequence $1.2n + 5$. (3mks)

Question Five (20 marks)

- (a) Find the truth values for the following statement, (5mks)

$$\left[(p \rightarrow q) \wedge (q \rightarrow r) \right] \Rightarrow (p \rightarrow r)$$

- (b) Prove validity of this statement. (5mks)

$$\begin{array}{l}
 1 \quad (K \Rightarrow A) \wedge (M \Rightarrow D) \\
 2 \quad \neg A \\
 3 \quad \neg D \quad / \quad \therefore \neg K \wedge \neg M
 \end{array}$$

- (c) Write the set $A = \{1, 4, 9, 16, 25, \dots\}$ in set-builder form. (2mks)
- (d) If $A = \{2, 4, 6, 8\}$ and $B = \{6, 8, 10, 12\}$. Find $A \cup B$ and $A \cap B$. (3mks)
- (e) Illustrate each of the following by shading the Venn diagrams below. (5mks)

