



KISII UNIVERSITY
UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF THE DEGREE
OF BACHELOR OF SCIENCE IN / ARTS IN EDUCATION
SECOND SEMESTER 2021/2022
JANUARY/MAY, 2022

MATH 111: CALCULUS 1

STREAM: Y1S2

TIME: 2 HOURS

March 21, 2022

INSTRUCTIONS

1. Do not write anything on this question paper.
2. Answer question ONE (*Compulsory*) and any other TWO questions

1 QUESTION ONE (30 MARKS)

a) Compute

i) $\lim_{x \rightarrow -2} x^3 - 2x + 1$

(2 mks)

ii) $\lim_{x \rightarrow 0} \frac{x^2}{4x - x^2}$

(2 mks)

iii) $\lim_{x \rightarrow \infty} \frac{x^3 + x - 20}{x^2 + 4x + 3}$

(2 mks)

b) Compute the following **implicit function**

i) $y^2 - 3yx = x^2$

(3 mks)

ii) $x^3 + 4xy = 15 + y^2$

(3 mks)

d) Find $\frac{dy}{dx}$ given the following $x = 2 + 3t$ when $y = \frac{1}{t}$

(3 mks)

e) Compute $y = x^3 \cos 3 \ln x$

(3 marks)

f) Differentiate the following polynomial function $y = x^3 + 8x^{\frac{1}{4}} - \frac{2}{\sqrt{x}}$

(3 mks)

g) Differentiate from first principle $y = x^3 + 5x + 10$

(3 mks)

h Investigate continuity of a function of intervals given

i) $f(x) = \begin{cases} x^2, & x < 2 \\ 4, & x \geq 0 \end{cases}$ on $[0, 4]$

(3 mks)

$$\text{ii) } f(x) = \begin{cases} \frac{x-6}{x-3}, & x < 0 \\ 2, & x = 0 \\ \sqrt{4+x^2}, & x > 0 \end{cases} \text{ at } x = 0$$

(3 mks)

2 QUESTION TWO (20 MARKS)

a) Use the quotient rule to differentiate

$$\text{i) } y = \frac{2e^t}{\sqrt{e^t + 4}}$$

(4 mks)

$$\text{ii) } y = \frac{3x}{(4x^2 - 1)}$$

(4 mks)

b) Use the product rule to differentiate

$$\text{i) } y = 2t \cdot \sqrt{3t^2 + 1}$$

(3 mks)

$$\text{ii) } y = \cos^3 x \sin^2 x$$

(3 mks)

c) Use the chain rule to differentiate

$$\text{i) } y = \sin(x^2 + 3x)$$

(3 mks)

d) Differentiate the following polynomial function

$$\text{i) } y = \frac{x^4 - x^3 + \sqrt{x} - 1}{x^2}$$

3 QUESTION THREE (20 marks)

a) Find $\frac{d^2y}{dx^2}$ given $x = (t^2 + 3)^{\frac{1}{2}}$ and $y = (3t + 4)^2$

(4 mks)

b) Differentiate

$$\text{i) } \sin^{-1} x$$

(3 mks)

$$\text{ii) } \cos^{-1} x$$

(3 mks)

iii) $\operatorname{cosec} x$

(3 mks)

c) Given $y = x^6 e^{2x}$ find $\frac{d^3 y}{dx^3}$

(3 mks)

d) Find $\frac{dy}{dx}$ given the following

$$x = \frac{t^2}{t+1} \text{ when } y = \frac{3t}{2-t}$$

(3 mks)

e) Given $y = 3x^2 - x$, show that $\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 6y + 1 = 6x$

(3 mks)

4 QUESTION FOUR (20 MARKS)

a) An average revenue (AR) and average cost (AC) functions for a given firm and products are

$$AR = 4 - \frac{1}{4}Q$$

$$AC = \frac{4}{q} + 2 - 0.3Q + 0.05Q^2$$

when q = number of units produced as sold

a) Find an expression for:

i) Marginal revenue ($M.R$)

ii) Marginal cost ($M.C$)

iii) Profit

b) Find

i) Number of units that will maximize profit and maximum profit

ii) Profit at unit maximum and maximum profit

(10 mks)

b) If $S = 4r^2 - 10r = 7$. Find maximum value of S and the value of r of which it is obtained

(3 mks)

c) Radius of circle increase at $2m/s$. Find rate at which area (A) is increasing in terms of r

(3 mks)

- d) Find equations of normal and tangents for curves given
 $3x^2 - 2xy + 2y^2 = 9$ at $(-2, -3)$

(4 mks)