Math 112-Basic Mathematics

November 23, 2021

Instructions Answer question one and any other two questions

Question one (30marks)

1a. Describe each of the following sets both in words and by listing out enough elements to see the pattern.

 i. $\{x \in \mathbb{N} : x + 3 \in \mathbb{N}\}$. **(2marks)**

 ii. $\{x^2 : x \in \mathbb{N}\}$. **(2marks)**

 b. Prove that $\sqrt{3}$ is an irrational number. **(3marks)**

 c. Given that $z_1 = 4 + 3i$ and $Z_2 = 2 + 3i$, find

 i. $Z_1 + Z_2$. **(2marks)**

 ii. $Z_1.Z_2$ **(2marks)**

 iii. $\dfrac{Z_1}{Z_2}$. **(2marks)**

 d. How many distinguishable arrangements are there of the word **SUCCESS**?. **(2marks)**

 e. By giving relevant examples define the following terms as used in set theory.

 i. empty set. **(2marks)**

 ii. universal set. **(2marks)**

 iii. equal sets. **(2marks)**

f. Simplify $Sin^2 x + Cos^2 x$ **(2marks)**

h. Let $A = \{1, 2, 3, 4\}$. Which ordered pairs are in the relation $R = \{(a, b)$:a *divides* b$\}$. **(3marks)**

i. If a club has **20** members, in how many different **5** member committees is possible?. **(2marks)**

j. Find the converse, opposite and contrapositive of the implication

If my car is not in the repair shop,then i cannot go to class. **(2marks)**

Question 2 (20marks)

2a. When is a relation R on a set A is said to be:

i. reflexive. **(1mark)**

ii. symmetric. **(1mark)**

iii. equivalence relation. **(1mark)**

iv. transitive. **(1mark)**

v. partial order. **(1mark)**

b. Show that the function $f : \mathbb{R} \longrightarrow (1, \infty)$ and $g : (1, \infty) \longrightarrow \mathbb{R}$ defined by $f(x) = 10x^2 + 1$ and $g(x) = \frac{1}{2}log_{10}(x - 1)$ are inverses of each other. **(6marks)**

c. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $g : \mathbb{R} \longrightarrow \mathbb{R}$ be defined by $f(x) = x^2$ and $g(x) = x + 5$. Find

i. $(g \circ f)(x)$. **(2mark)**

ii. $(f \circ g)(x)$. **(2mark)**

Question 3 (20 marks)

3a. Show that $1 + sin2x = (sinx + cosx)^2$. **(3marks)**

b. Simplify

i. $Sin^2 x Cos^2 x + Cos^4 x$. **(2marks)**

ii. $\dfrac{1 - Cos^2x - Sinx}{2Sin^2x + Sinx - 3}$. **(3marks)**

b. Solve the following trigometric equations

i $12cos^2x + sinx = 11$ on the domain $0° \leqq x \geq 360°$. **(3marks)**

ii. $sin^2x - cos^2x = -\dfrac{1}{2}$ for $0° \leqq x \geq 360°$. **(3marks)**

c. Express each of the following as a sum or difference

i. Sin $40°$ cos $30°$. **(2marks)**

ii. cos$110°$ sin$55°$. **(2marks)**

d. Find an equivalent expression for $Sin3\theta$ interms of θ. **(2marks)**

Question 4 (20marks)

a. Distinguish between permutations and combinations. **(2marks)**

b. How many committees of 5 people can be chosen from 20 men and 12 women

i. If exactly 3men must be on each committees. **(2marks)**

ii. If atleast 4 women must be on each committes. **(2marks)**

c.i State Binomial theorem. **(2marks)**

ii. Use binomial theorem to prove that

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots \binom{n}{n} = 2^n$$

. **(3marks)**

d. Use venn diagram to prove that $A - B = A \cap B^c$. **(2marks)**

e. List all the subset of the set $S = \{a, b, c\}$. **(2marks)**

f. Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6\}$ and $\{1, 2, 3\}$.If the universal set $U = \{1, 2, 3, 4 \cdots 10, \}$ find

i $A \cup B$. **(2marks)**

ii $\overline{B \cup C}$. **(2marks)**

iii. $\phi \cap C$. **(2marks)**

Question 5(20marks)

4a. Indicate whether each of the following statements is a proposition or not. State state the truth values for the propositions.

  i. When will you come to Kenya?. **(1mark)**

  ii. Naivasha is a county in Kenya. **(1mark)**

  iii. Tomorrow is my birthday. **(1mark)**

  iv. 51 is a prime number. **(1mark)**

b.i Differentiate between tautology and contradiction. **(2marks)**

  ii. Show that $P \lor \sim P$ is a tautology and $P \land \sim P$ is a contradiction. **(4marks)**

  c. Construct truth table for the following logic connectives:

  i. $P \land Q$. **(2marks)**

  ii. $P \Rightarrow Q$. **(2marks)**

  iii. $P \Leftrightarrow Q$. **(2marks)**

  c. Prove by Mathematical induction that $1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$ for all $n \geq 1$. **(4marks)**