KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED STATISTICS

SECOND SEMESTER 2021/2022
(FEBRUARY-JUNE, 2022)

MATH 116: MATRIX ALGEBRA

STREAM: Y1 S2 **TIME: 2 HOURS**

DAY: MONDAY, 9:00 AM – 11:00 AM **DATE: 30/05/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
   ***Answer Question ONE (Compulsory) and any other TWO* Questions.**

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Calculate the determinant of the following matrices:

i) $A = \begin{pmatrix} \frac{1}{2} & \frac{2}{3} \\ \frac{1}{3} & -\frac{3}{5} \end{pmatrix}$ ii) $A = \begin{bmatrix} 1 & 5 & -2 \\ 3 & -1 & 4 \\ -3 & 6 & -7 \end{bmatrix}$ (6 marks)

b) Given the matrices $A = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$, determine $(AB)^{-1}$

(7 marks)

c) Use matrices to solve:

i) $12u + 8a = 52$
$-16u + 6a = -36$ (3 marks)

ii) $a + 2b - 3c = 3$

$$2a - b - c = 11$$

$$3a + 2b + c = -5$$ (5 marks)

d) Given the matrix $A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$ and that $A^2 - 5A + kI = 0$, where k is a constant, determine the value of k. (5marks)

e) Use Cramer's rule to solve:

$$2I_1 + 3I_2 - 4I_3 = 26$$
$$I_1 - 5I_2 - 3I_3 = -87$$
$$-7I_1 + 2I_2 + 6I_3 = 12$$

(4 marks)

QUESTION TWO (20MARKS)

a) Use Gauss-Elimination method to solve:

$$x + y + z = 4$$
$$2x - 3y + 4z = 33$$
$$3x - 2y - 2z = 2$$ (6mks)

b) Find the eigenvalues of the matrix:

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$ (7 marks)

c) Use the method of determinants to solve the simultaneous equations.

$$7x - 4y = 2$$
$$-4x + 5y - 3z = 10$$
$$-3y + 52z = -14$$ (7mks)

QUESTION THREE (20MARKS)

a) Find the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Hence find A^{-1}. (7 marks)

b) Solve the equation $\begin{vmatrix} 4 & -x \\ 5 & 2x \end{vmatrix} = 4$ (3 marks)

c) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$ (10 marks)

QUESTION FOUR (20MARKS)

Find the modal matrix P and the resulting diagonal matrix D of A, if:

$$A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$$ (20 marks)

QUESTION FIVE (20MARKS)

a) Reduce quadratic form to canonical form using orthogonal transformation.

$$3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_3x_1 - 2x_1x_2$$

(12 marks)

b) The relationship between the displacement, s, velocity, v, and acceleration, a, of a piston is given by the equations:

$$3s + 2v - 2a = 32$$
$$4s + 3v + 3a = 4$$
$$-2s + v - a = 2$$

Use matrices to determine the values of s, v and a. (8 marks)