KISII UNIVERSITY
SCHOOL PURE AND APPLIED SCIENCES
DEPARTMENT MATHEMATICS AND ACTUARIAL SCIENCE
COURSE TITLE:FUNDAMENTAL ACTUARIAL MATHEMATICS II
COURSE CODE: BACS 210
FINAL EXAM JUNE 2022
INSTRUCTIONS:Answer question one and any other 2 questions in section B

SECTION A (30 marks)

Question One (30 marks)

(a) The tables below show cumulative data for the number of claims and the total claim amounts arising from a portfolio of insurance policies.

	Claim Numbers Development Year		
	0	*1*	*2*
2010	87	132	151
2011	117	156	
2012	99		

	Total Claim Amounts Development Year		
	0	*1*	*2*
2010	43,290	87,430	126,310
2011	68,900	125,290	
2012	74,250		

Claims are fully run off after two development years. Estimate the outstanding claims using the average cost per claim method with grossing up factors. (7mks)

(b) An actuarial student has been working on some claims projections but some of her workings have been lost. The cumulative claim amounts and projected ultimate

Accident Year	*Development Year* 0	1	2	3	*Ultimate*
1	1001	1485	1762	*W*	*X*
2	1250	*Y*	1820		1862.3
3	1302	1805			2122.5
4	*Z*				2278.8

claims are given by the following table:
All claims are paid by the end of development year 3. It is known that ultimate claims for accident years 2 and 3 have been estimated using the Basic Chain Ladder method. Calculate the values of W, X and Y. (3mks)

(c) Calculate the single premium payable for a temporary reversionary annuity of 12,000 per annum payable monthly in arrear to a female life currently aged 55 exact on the death of a male life currently aged 50 exact. No payment is made after 20 years from the date of purchase. (5mks)

(d) A university student receives a 3-year sponsorship grant. The payments under the grant is as follows:Year 1 5,000 per annum paid continuously, Year 2 5,000 per annum paid monthly in advance and Year 3 5,000 per annum paid half yearly in advance. Calculate the total present value of these payments at the beginning of the first year using a rate of interest of 0.08 p.a convertible quarterly. (5mks)

(e) The table below shows the cumulative incurred claim amounts on a portfolio of insurance policies.

Underwriting Year	*Development Year* 0	1	2
2014	3,215	6,847	10,078
2015	2,986	7,123	
2016	4,167		

Claims are assumed to fully run off after Development Year 2. The estimated loss ratio of both 2015 and 2016 is 91% and the respective premium income in each year is:

	Premium Income
2014	11,365
2015	12,012
2016	12,867

The total of claim amounts paid to date is 21,186 from policies written in 2014 to 2016. Calculate the outstanding claim reserve for this portfolio using the Bornheutter-Ferguson method. (7mks)

(f) Define ${}_{4|5}q_{[60]+1}$ and calculate its value (3mks)

SECTION B

Question Two (20 marks)

(a) The run-off triangle below shows cumulative claims incurred on a portfolio of general insurance policies.

Accident year	*Development year*		
	0	*1*	*2*
2017	2,440	3,294	3,788
2018	2,065	2,849	
2019	2,158		

Past and projected future inflation is given by the following index (measured at the midpoint of the relevant year).

Year	*Index*
2017	100
2018	105
2019	109
2020	116
2021	123

Calculate the outstanding claims using the inflation adjusted chain ladder method. (6mks)

(b) A life aged exactly 50 buys a 15-year endowment assurance policy with a sum assured of 50,000 payable on maturity or at the end of the year of earlier death. Level premiums are payable monthly in advance. Calculate the monthly premium assuming AM92 Ultimate mortality and 4% pa interest. Ignore expenses. (4mks)

(c) For the first 5 years after arrival in a certain country, lives are subject to a constant force of mortality of 0.005. Thereafter lives are subject to mortality according to English Life Table No. 12- Males with an addition of 0.039221 to the force of mortality. A life aged exactly 30 has just arrived in the country calculate the probability that it will survive to age 35. (5mks)

(d) A whole life annuity is payable continuously to a life now aged 60. The rate of payment at time t is:

$$\rho(t) = 10{,}000(1.02)^t \qquad (t > 0)$$

Write down an expression for the present value of the annuity in terms of annuities-certain and expressions for the expected present value and variance of the present value of the annuity. (5mks)

Question Three (20 marks)

(a) The table below shows incremental claim amounts paid on a portfolio of general insurance policies, where claims are assumed to fully run off after three years.

Underwriting Year	*Development Year* 0	1	2	3
2012	504	286	110	35
2013	621	302	120	
2014	685	340		
2015	801			

Past and projected future inflation is given by the following index (measured to the mid-point of the relevant year).

Year	*Index*
2012	100
2013	103
2014	105
2015	106
2016	105
2017	107
2018	110

Estimate the outstanding claims reserve using the inflation-adjusted chain ladder technique. (5mks)

(b) The mortality of a certain population is governed by the life table function $l_x = 100 - x, \;\; 0 \leq x \leq 100$. Calculate the values of the following expressions: ${}_{10}p_{30}$ and μ_{30} (5mks)

(c) An annuity provides payments of 40 at the end of each month forever. If the interest rate is 10 percent pa convertible quarterly, calculate the present value of the annuity. (5mks)

(d) A life aged 65 is assumed to be subject to an annual initial rate of mortality equal to twice that of the *AM*92 Ultimate tables for the next two years. Calculate the probability that the life will die before age 67. (5mks)

Question Four (20 marks)

(a) In its premium rate basis, an office assumes a 3-year select period. The table is such that: $q_{[x+7]} = q_{([x+3]+1)} = q_{([x]+2)} = q_{(x+1)}$ and ultimate mortality follows A1967-70 ultimate. Calculate $l_{[x]+t}$ for $t = 0, 1$ and 2. (5mks)

(b) The table below shows the cumulative incurred claims on a portfolio of insurance policies.

	Development Year		
Accident Year	*1*	*2*	*3*
2016	130	180	190
2017	140	185	
2018	150		

The company decides to apply the Bornhuetter-Ferguson method to calculate the reserves, with the assumption that the Ultimate Loss Ratio is 80%. Claims are assumed to be fully run off by development year 3. The earned premium for 2018 is 300 and the paid claims for 2018 are 100. Calculate the reserve in respect of the accident year 2018. (5mks)

(c) A life aged exactly 60 wishes to arrange for a payment to be made to a charity in 10 years' time. If he is still alive at that date the payment will be 1000. If he dies before the payment date, the amount given will be 500. Assuming an effective interest rate of 6% per annum and mortality according to ELT No.12-Males, calculate the standard deviation of the present value of the liability. (5mks)

(d) A life aged 40 effects a 25-year without profits endowment assurance policy with a sum assured of 50,000 (payable at the end of the year of death or on survival to the end of the term). Level premiums are payable annually in advance throughout the term of the policy or until earlier death of the life assured. Calculate the level premium, P, using the following premium basis Mortality: A1967-70 Ultimate; Interest: 6% p.a. Expenses: none (5mks)

Question Five (20 marks)

(a) The claims paid to date on a motor insurance policy are as follows (Figures in 000s):

	Development year			
Policy Year	*0*	*1*	*2*	*3*
2001	1,256	945	631	378
2002	1,439	1,072	723	
2003	1,543	1,133		
2004	1,480			

Inflation for the 12 months to the middle of each year was as follows:

2002	2.10%
2003	1.20%
2004	–0.80%

Annual premiums written in 2004 were 5,250,000. Future inflation from mid-2004 is estimated to be 2.5% per annum. The ultimate loss ratio (based on mid-2004 prices) has been estimated at 75%. Claims are assumed to be fully run-off at the end of development year 3. Estimate the outstanding claims arising from policies written in 2004 only (taking explicit account of the inflation statistics in both cases), using:

(i) Calculate the reserve for unpaid claims using the inflation-adjusted chain ladder method. (5mks)

(b) Find expressions in terms of the life table functions $l_{[x]+t}$ and l_y for $q_{[50]}$, ${}_{2|}q_{[50]}$, ${}_{2|3}q_{[50]+1}$ (5mks)

(c) Calculate the annual premium payable by a man aged 32 for a temporary assurance with a sum assured of $5,000$ and a term of 12 years. Assume AM92 Ultimate mortality and 4% pa interest. (5mks)

(d) If ${}_nq_x = 0.3$ and ${}_nq_y = 0.5$ calculate ${}_nq_{xy}$ and ${}_nq_{\bar{xy}}$ (5mks)