KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE, EDUCATION
FIRST SEMESTER 2021/2022
(FEBRUARY-JUNE, 2022)

MATH 210: LINEAR ALGEBRA I

STREAM: Y2 S1 **TIME: 2 HOURS**

DAY: TUESDAY, 9:00 AM – 11:00 AM **DATE: 10/05/2022**

INSTRUCTIONS:

1. Do not write anything on this question paper.
***2. Answer Question ONE (Compulsory) and any other TWO* Questions**

QUESTION ONE (30 MARKS)

a. Consider the following systems of linear equation: (3 marks)

$$2x + 4y = 8$$
$$4x - 2y = 6$$
$$6x + 2y = 2k$$

Find the value of k for which the system is consistent

b. Use Gauss-Jordan Elimination method to solve the following systems of linear equations. (6 marks)

$$3x_1 + 3x_2 + 6x_3 = 24$$
$$3x_1 + 6x_2 - 9x_3 = -3$$
$$9x_1 - 21x_2 + 12x_3 = 30$$

c. Find the values of λ for which the determinant of the matrix below is equal to zero. (5 marks)

$$\begin{bmatrix} \lambda + 1 & 0 & 0 \\ 4 & \lambda & 3 \\ 2 & 8 & \lambda + 5 \end{bmatrix}$$

d. Let

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & -3 & 10 \\ 0 & 1 & -3 \end{bmatrix}$$

Find the ad joint of A and hence find A^{-1} (8 marks)

e. i.) State the Cramer's rule (2 marks)

ii.) Use Gaussian elimination to solve the following systems of linear equations. (6 marks)

$$x_1 + 2x_2 + 3x_3 = 9$$
$$-x_1 + 3x_2 = -4$$
$$2x_1 - 5x_2 + 5x_3 = 17$$

QUESTION TWO (20 MARKS)

a. i.) Define the term symmetric matrix. (1 mark)

ii.) Prove that a symmetric matrix of order 2 is diagonalizable. (4 marks)

iii.) State the Cayley – Hamilton's theorem and use it to verify for the matrix. (4 marks)

$$A = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$$

b. Show that the determinant of a second order matrix with identical rows is zero. (2 marks)

c. Consider the matrices $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -4 \\ 4 & -1 \end{bmatrix}$ determine whether these matrices commute and hence find the commutator. (4 marks)

d. Use Cramer's rule to find the point of intersection of the three planes defined by: (5 marks)

$$x + 2y - z = 4$$
$$2x - 2y + 3z = 3$$
$$4x + 3y - 2z = 5$$

QUESTION THREE (20 MARKS)

a. Apply the Gram-Schmidt process to construct an orthogonal basis set for (7 marks)

$$B = \{(1,1,0), (1,2,0), (0,1,2)\} \; of \; \mathbb{R}^3$$

b. Show that the transformation $T(x) = 2x + 1$ is not a linear transformation. (3 marks)

c. Find the Eigen values of the matrix $A = \begin{bmatrix} 5 & 2 \\ 9 & 2 \end{bmatrix}$ (5 marks)

d. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ Determine whether or not A is diagnosable. (5 marks)

QUESTION FOUR (20 MARKS)

a. Calculate the area of the triangle whose vertices are $A(1,0), B(2,2) and\ C(4,3)$ by use of the method of determinants. (4 marks)

b. Consider the vectors $u = (1, -3, 7) and\ v = (8, -2, -2)$ Find u, v and the angle between them

(5mks)

c. i.) Let $V = \mathbb{R}^3$ with standard operations and $S = \{(1,2,3), (0,1,2), (-2,0,1)\} \leq \mathbb{R}^3$
Does SSpan V?

ii.) Let $V = \mathbb{R}^3$ and $S = \{(-4, -3, 4), (1, -2, 3), (6,0,0)\}$ determine whether S is linearly independent (4 marks)

d. Find the rank of the matrix below (4 marks)

$$A \begin{bmatrix} 1 & -2 & 3 \\ 2 & -5 & 1 \\ 1 & -4 & -7 \end{bmatrix}$$

QUESTION FIVE (20 MARKS)

a. i.) Define the term Basis of a vector space V (2 marks)

ii.) Let $S = \{V_1 = (1,2,1), V_2 = (2,9,0), and\ V_3 = (3,3,4)\}$ (3 marks)
Show that the set S is basis for $\mathbb{R}^3$

b. Show that $< u, v >= u_1 v_1 - 2u_2 v_2\ where\ u = (u_1 u_2)\ v = (v_1 v_2)$ is an inner product on $\mathbb{R}^2$ (5mks)

c. i.) Define the term Linear transformation. (1 mark)

ii.) Show that $T: \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_2 - x_1 \end{bmatrix}$ is a linear transformation (6 marks)