KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

SECOND SEMESTER 2021/2022

(FEBRUARY-JUNE, 2022)

MATH 220: VECTOR ANALYSIS

STREAM: Y2 S2 **TIME: 2 HOURS**

DAY: FRIDAY, 12:00 PM – 2:00 PM **DATE: 13/05/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer Question ONE (Compulsory) and any other TWO* Questions.**

QUESTION ONE (COMPULSORY) 30MKS

a) Show that $\frac{d}{dt}(\boldsymbol{F} \times \boldsymbol{G}) = \boldsymbol{F} \times \frac{d\boldsymbol{G}}{dt} + \frac{d\boldsymbol{F}}{dt} \times \boldsymbol{G}$ (3 marks)

b) Determine the curl of vector $\vec{F}$ at the point (2, 0, 3) given that $\vec{F} = \mathbf{i}ze^{xy} + \mathbf{j}2xz\cos y + (x + 2y)\mathbf{k}$. (4 marks)

c) Find the scalar triple product of $2\mathbf{i} + \mathbf{k}, \mathbf{j} - 3\mathbf{k}, \mathbf{j} + 2\mathbf{i}$ (3 marks)

d) Show that $V = (x + 3y)\mathbf{i} + (y - 3z)\mathbf{j} + z\mathbf{k}$ is solenoidal (3 marks)

e) If $\vec{A} = x^2z\mathbf{i} + xy\mathbf{j} + y^2z\mathbf{k}$ and $\vec{B} = yz^2\mathbf{i} + xz\mathbf{j} + x^2z\mathbf{k}$. Determine the expression for grad $(\vec{A}.\vec{B})$ at (1,1,1) (4 marks)

f) A particle moves in space so that at time t its position is stated as $x = 2t^2, y = 2t^2 - 4t, z = 3t - 5$. Find the component of velocity in the direction $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ (4 marks)

g) If If $\vec{F} = x^2y^2\mathbf{i} + y^3z\mathbf{j} + z^2\mathbf{k}$. Evaluate $\int_{\mathbf{c}} \vec{F}.dr$ along the curve $x = 2u^2,\ y = 3u$ and $z = u^3$ between $A(2, -3, -1)$ and $B(2, 3, 1)$ (5 marks)

h) Find the directional derivative of $\emptyset = z^3y + y^2x$ at the point (1,-1,1) in the direction of the vector $\mathbf{a} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$ (4 marks)

QUESTION TWO (20 MARKS)

a) If $\vec{F} = 2z\mathbf{i} - x\mathbf{j} + y\mathbf{k}$, evaluate $\iiint_V \vec{F}dv$ where V is the region bounded by the surfaces $x = 0, x = 2, y = 0, y = 4, z = x^2, z = 2$. (6 marks)

b) Use divergence theorem to evaluate the $\iint \vec{F}.\hat{n}\ ds$ where $\vec{F} = (3x - 2z)\mathbf{i} - (2x + y)\mathbf{j} + (y^2 + 2z)\mathbf{k}$ and S is the surface of the sphere with centre at (1,2,4) radius 4 units

(8 marks)

c) Prove that $(y^2 - z^2 + 3yz - 2x)\mathbf{i} + (3xz + 2xy)\mathbf{j} + (3xy - 2xz + 2z)\mathbf{k}$ is both solenoidal and irrotational. (6 marks)

QUESTION THREE (20 MARKS)

a) A fluid motion is given by $\vec{v} = (y + z)\mathbf{i} + (z + x)\mathbf{j} + (x + y)\mathbf{k}$.Show that the motion is irrotational and hence find the velocity potential. (8 marks)

b) If $\vec{A} = t^2\mathbf{i} - t\mathbf{j} + (2t + 1)\mathbf{k}$and $\vec{B} = (2t - 3)\mathbf{i} + \mathbf{j} - t\mathbf{k}$. Find $\frac{\partial}{\partial t}(\vec{A} \times \vec{B})$ at t = 1 (6 marks)

c) Given that $u = x^2 + y^2 + z^2$ and $\vec{r} = xi + yj + zk$ find div u**r** (6 marks)

QUESTION FOUR (20 MARKS)

a) The acceleration of a particle at any time t ≥ 0 is given by $\vec{a} = e^{-t}\mathbf{i} - 6(t + 1)\mathbf{j} + 3\sin(t)\mathbf{k}$. If the velocity $\vec{v}$ and displacement $\vec{r}$ are zero at t = 0. Find $\vec{v}$ and $\vec{r}$ at any time. (9 marks)

b) If a force $\vec{F} = 2x^2y\mathbf{i} + 3xy\mathbf{j}$ displaces a aparticle in the x-y plane from (0,0) to (1,4) along a curve $y = x^2$. Find the work done. (6 marks)

c) A scalar function φ is given as $\varphi = \frac{\vec{r}}{|r|^3}$ where $\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Find $\nabla\varphi$ (5 marks)

QUESTION FIVE (20 MARKS)

a) i) state stokes theorem (2 marks)

ii) Use stock's theorem to evaluate $\int[(2x - y)dx - (yz^2)dy - (y^2z)dz]$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is boundary of the sphere. (6 marks)

b) Use green's theorem to compute $\oint_c xy\ dx + xy\ dy$ over a counterclockwise rectangle with corners at (1, 0), (3, 0), (3, 2) and (1, 2). (6 marks)

c) If $\vec{F} = (x^2y)\mathbf{i} + 2yz\mathbf{j} + 3xz^2\mathbf{k}$. Evaluate $\int_c \vec{F}.d\mathbf{r}$ from (0,0,0) to (1,2,3) along the path given by x = t , y = 2t and z = 3t (6 marks)