KISII UNIVERSITY
UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE ACTURIAL SCIENCE
SECOND SEMESTER 2021/2022
(FEBRUARY-JUNE, 2022)

BACS 301 - RISK THEORY FOR ACTUARIAL SCIENCE

STREAM: Y3 S2 **TIME: 2 HOURS**

DAY: WEDNESDAY, 12:00 PM – 2:00 PM **DATE: 25/05/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer Question ONE (Compulsory) and any other TWO* Questions.**

QUESTION ONE (30 marks)

1. List five risk criteria that would be considered desirable by a general insurer. (4marks)
2. Claims on a group of policies of a certain type arise as a Poisson process with parameter μ1 Claims on a second, independent, group of policies arise as a Poisson process with parameter μ2. The aggregate claim amounts on the respective groups are denoted *Y*1 and *Y*2. Using MGFs show that *Y* (the sum of *Y*1 and *Y*2) also has a compound Poisson distribution and hence derive the Poisson parameter for *Y*. (5marks)
3. A compound random variable $Y = Z_1 + Z_2 + \cdots + Z_M$ has claim number distribution: $P(M = m) = 9(m + 1)4^{-m-2}, m = 0,1,2, \ldots$ The individual claim size random variable, *Z,* is exponentially distributed with mean 2.Calculate *E*(*Y*) and var(*Y*). (5marks)
4. Write down a formula for the MGF of a compound Poisson distribution with individual claim size distribution *Gamma* (a, b) and Poisson parameter μ. *(5marks)*
5. The distribution of the number of claims from a motor portfolio is NB (5000,0.8). The claim size distribution is Pareto (4,1500). Calculate the SD of the aggregate claim distribution. (6marks)

6. Write the R-Code used to simulate 400 values from a NB(5000,0.8) and a Pareto(4,1500) claims distribution and hence generate the mean, SD, coefficient of skewness, P(S>200), 90^{TH} Percentile. (5marks)

SECTION B QUESTION 2 (20MARKS)

1. Let $Y_1 + Y_2 + \cdots + Y_n$ be independent random variables. Suppose that each Y_i has a CPD with parameter μ_i and that the CDF of each individual claim amount for each Y_i is $F_i(x)$show that $Z = Y_1 + Y_2 + \cdots + Y_n$ is also CPD using MGFs (6marks*)*

2. Write the R-Code to simulate 500 values for a reinsurer where claims have a CPD with parameter 10 and an exponential (0.7) claims distribution under excess of loss with retention 25 (6marks)

3. The number of claims from a given portfolio has a Poisson distribution with a mean of 2.5 per month. Individual claim amounts have the following distribution:
   Amount 200 300
   Probability 0.55 0.45
   An aggregate reinsurance contract has been arranged so that the insurer pays no more than 400 per month in total. Assuming that the individual claim amounts are independent of each other and are also independent of the number of claims, calculate the expected aggregate monthly claim amounts for the insurer and the reinsurer. (8marks)

QUESTION 3 (20MARKS)

1. The annual aggregate claim amount from a risk has a CPD with Poisson parameter 100. Individual claim amounts are U (0, 20000). The insurer of this risk has effected excess of loss reinsurance with retention level 16000. Calculate the coefficient of skewness of both the insurer and reinsurers aggregate claims under this reinsurance agreement. (7marks)

2. Suppose the poisson parameters for individual policies are drawn from a gamma distribution with parameters a and d. Find the distribution of the number of claims from a policy chosen at random from the portfolio. (6marks)

3. A group of policies can give rise to at most 2 claims. The probabilities of 0, 1 or 2 claims are ½, ¼ and ¼ respectively. Claim amounts are IID *U* (0, 10) random variables. Let *Y* denote the aggregate claim amount random variable. Sketch the frequency distribution of *Y*. (7marks)

QUESTION 4 (20MARKS)

1. Show that the compound binomial distribution becomes negatively skewed if $p > 0.5$ and all claims are of a similar amount. (10marks)

2. An insurance company offers accident insurance for employees. A total of 650 policies have been issued split between two categories of employees. The first category contains 400 policies, and claims occur on each policy according to a Poisson process at a rate of one claim per 20 years, on average. In this category all claim amounts are £3,000. In the second category, claims occur on each policy according to a Poisson process at a rate of one claim per 10 years, on average. In this category, the claim amount is either £2,000 or £3,000 with probabilities 0.3 and 0.7, respectively.All policies are assumed to be independent. Let *Y* denote the aggregate annual claims from the portfolio.
(i) Calculate the mean, variance and coefficient of skewness of *Y*.
(ii) Using the normal distribution as an approximation to the distribution of *Y*, calculate *W* such that the probability of *Y* exceeding *W*is 10%.
(iii) The insurance company decides to effect reinsurance cover with aggregate retention£100,000, so that the insurance company then pays out no more than this amount inclaims each year. In the year following the inception of this reinsurance, the numbers ofpolicies in each of the two groups remains the same but, because of changes in theemployment conditions of which the company was unaware, the probability of a claim ingroup 2 falls to zero. Using the normal distribution as an approximation to thedistribution of *S* , calculate the probability of a claim being made on the reinsurancetreaty. (10marks)

QUESTION 5 (20MARKS)

1. A group of policies can give rise to at most two claims in a year. The probability function for the number of claims is as follows:

Number of claims, m	0	1	2
$P(M = m)$	0.5	0.4	0.1

Each claim is either for an amount of 1 or an amount of 2, with probabilities 0.6 and 0.4 respectively. Claim amounts (Z) are independent of one another and are independent of the number of claims (M). Determine the distribution function of the aggregate annual claim amount (*Y*). (7marks)

2. A portfolio consists of 5000 independent risks. For the *i* th risk, with probability 1-*qi* there are no claims in one year, and with probability *qi* there is exactly one claim (0<*qi* <1). For all risks, if there is a claim, it has mean μ, variance θ2 and moment generating function *M*(*t*) . Let *Y* be the total amount claimed on the whole portfolio in one year.

i) Determine the mean and variance of Y. (5marks)

The amount claimed in one year on risk i is approximated by a compound Poisson random variable with Poisson parameter qi and claims with the same mean μ, the same variance θ2, and the same moment generating function $M(t)$ as above.

Let $\hat{Y}$ denote the total amount claimed on the whole portfolio in one year in this approximate model.

ii) Determine the mean and variance of $\hat{Y}$, and compare your answers to those in part (i). (4marks)

Assume that $qi = 0.02$ for all i , and if a claim occurs, it is of size μ with probability one.

iii) Derive the moment generating function of $\hat{Y}$ and show that Y has a compound binomial distribution. (2marks)

iv) Determine the moment generating function of the approximating $\hat{Y}$, and show that $\hat{Y}$ has a compound Poisson distribution. (2marks)