KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

MAIN CAMPUS

FIRST YEAR EXAMINATIONS FOR THE AWARD OF

THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

SECOND SEMESTER 2021/2022

(May August 2022)

PHYS 817: STATISTICAL MECHANICS

Instructions

1) Attempt question **one** and any other **three** questions
2) All question have the same marks
3) Clearly show your work and justify your answers

Question One [15 marks]

Consider the classical ideal gas. The total entropy could be written as:

$$\mathbf{S (E, V, N) = (N/N_o)S_o + Nk_B \ln [(E/E_o)^{3/2}(V/Vo)(N/No)^{-5/2}]}$$

Where E is the total internal energy, V is the total volume, and N is the number of particles. $\mathbf{E_o}$, $\mathbf{V_o}$, $\mathbf{N_o}$, $\mathbf{S_o}$, and k_B are constants.

(a) Starting from the above S(E, V, N), find the Helmholtz free energy A(T, V, N), the Gibbs free energy G(T, p, N), and the Grand Potential Φ(T, V, μ), by the method of Legendre transforms. [10 points]

(b) Find the familiar equation of state, pV = NkBT, by taking an appropriate 1st derivative of an appropriate thermodynamic potential. [5 points]

(c) Find the chemical potential μ, by taking an appropriate first derivative of a thermodynamic potential. By comparing to your result in part (a), show explicitly that the chemical potential is the same as the Gibbs free energy per particle. [5 points]

(d) Find the pressure p, by taking an appropriate first derivative of a thermodynamic potential. By comparing to your result in part (a), show explicitly that the pressure is the same as the negative of the Grand Potential per volume. [5 points]

Question Two [20 marks]

Consider a box of volume V. The box is split exactly in half by a thermally conducting imovable wall. Equal quantities of the same type of ideal gas fill each half of the box, and the system is in equilibrium. The total energy of the gas is fixed at ET.

a) Show that the pressure of the gas on each side is the same (remember, the wall is imovable, so you can't just appeal to mechanical equilibrium).

b) Using the formular derived in lecture for the number of states Ω(E) of an ideal gas at total energy E, find the most likely value for the energy of the gas on one side of the box.

c) Using Ω(E), if P(E) is the probability distribution for the gas on one side of the box to have energy E, show that the relative width (i.e. the width divided by the average) of P(E) is proportional to 1/N1/2, for sufficiently large N, where N is the number of particles in the gas. This shows that the fluctuation of E away from its average becomes negligibly small in the thermodynamic limit of N going to infinity. (For "width" you may use half width at half height or any other reasonable definition.)

Question Three [20 marks]

In a particular engine a gas is compressed in the initial stroke of the piston. Measurements of the instantaneous temperature, carried out during the compression, reveal that the temperature increases according to the relation:

$T = (V/Vo)^{\eta}T_o$

where To and Vo are the initial temperature and volume and η is a constant. The gas is compressed to the volume V1 (where V1 < Vo). Assume that the gas is a monatomic ideal gas of N atoms, and assume the process is quasi-static (i.e. the system is always instantaneously in equilibrium).

a) Calculate the mechanical work done on the gas. [5 points]

b) Calculate the change in the total energy of the gas. [5 points]

c) Calculate the heat transfer Q to the gas. For what value of η is Q = 0? Show that this corresponds to the case of adiabatic compression (see this week's discussion question, above). [5 points]

[Hint: you may use the facts you know about an ideal gas, i.e. $pV = Nk_BT$, and $E = (3/2)Nk_BT$.]

Question Four [20 marks]

Taking 2nd derivatives of the appropriate thermodynamic potentials for the ideal gas, compute the specific heats C_V and Cp, the compressibility κT and κS, and the coefficient of thermal expansion α. Show by direct comparison of these results that the two specific heats, and the two compressibilities, are indeed related to each other by the general formulae

Question Five [20 marks]

Consider a system of N distinguishable non-interacting objects, each of which can be in one of two possible states, "up" and "down", with energies +ε and −ε. Assume that N is large.

(a) Working in the micro canonical ensemble, find the number of states Ω(E,N), and then the entropy of the system S(E, N), as a function of fixed total energy E and number N (Hint: it may be useful to consider the numbers N+ and N− of up and down objects). Sketch S(E,N) as a function of E for fixed N, and show that it is not a monotonic increasing function of E. What is the key feature of this system that causes the dependence of S on E to be so qualitatively different from that of an ideal gas? [10 points]

(b) Using your result from (a) find the temperature T as a function of energy E and number N. Show that T will be negative if E>0. Sketch T vs E for fixed N. [5 points]

(c) What happens if such a system (1) with T1<0 comes into thermal contact with another such system (2) with T2>0? Does T1 increase or decrease? Does T2 increase or decrease? In which direction does the heat flow? [5 points]