KISII UNIVERSITY
UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF MASTER OF SCIENCE IN PHYSICS
SECOND SEMESTER 2021/2022
[JULY, 2022]

PHYS 823: COMPUTATIONAL PHYSICS

STREAM: Y1 S2 **TIME: 3 HOURS**

DAY: TUESDAY, 2.00 PM –5.00 PM **DATE: 05/07/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer Question ONE (Compulsory) and Any Other TWO Questions.***

QUESTION ONE [30 MARKS]

a) Exactly what will be displayed after the following MATLAB commands are typed? [2marks]

(i)
```
>> x = 5;
>>x ^ 3;
>> y = 8 – x
```

(ii)
```
>> q = 4:2:12;
>> r = [7 8 4; 3 6 –5];
>>sum(q) * r(2, 3)
```

b) The butterfly curveis given by the following parametric equations:

$$x = \sin(t)\left(e^{\cos t} - 2\cos 4t - \sin^5 \frac{t}{12}\right)$$

$$y = \cos(t)\left(e^{\cos t} - 2\cos 4t - \sin^5 \frac{t}{12}\right)$$

Write a Matlab code to generate values of xand yfor values of tfrom 0 to 100 with $\Delta t = 1/16$ hence a plot of

i. xand yversus t [3marks]

ii. yversus x. [2marks]

Include titles and axis labels onboth plots and a legend for (i). For (i), employ a dotted linefor yin order to distinguish it from x.

c) Show how the following integral can be evaluated in Matlabsymbolically. [2marks]

$$\int_0^{\pi/2} (8 + 4\cos x)\, dx$$

d) Use Euler method of $h = 0.5$ to solve the following initial value problem over theinterval from t =0to 2 where y(0)=1. [4marks]

$$\frac{dy}{dt} = yt^3 - 1.5y$$

e) From Taylor series expansion, derive the centered difference approximation of derivatives. [3marks]

f) Based of the equation derived in (e) above, develop an m file that will implement this approximation. [5marks]

g) The sine function can be evaluated by the following infinite series:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ggg$$

Create an M-file to implement this formula so that it computes and displays the values of sin x as each term in the series is added. [5marks]

h) Distinguish between a **while loop** and a **for loop**. [2marks]

i) Explain the purpose of the following matlab codes: **clc, close all, format long and xlsread** [2marks]

QUESTION TWO [15 MARKS]

a) What is an M file? Name one type. [1marks]

b) An amount of money Pis invested in an account where interest is compounded at the end of the period. The future worth F yielded at an interest rate i after nperiods may be determined from the following formula:

$$F = P(1+i)^n$$

Write an M-file that will calculate the future worth of an investment for each year from 1 through n. The input to thefunction should include the initial investment P, the interestrate i(as a decimal), and the number of years nfor which thefuture worth is to be calculated. The output should consist ofa table with headings and columns for n and F. Show how your program will be called to compute the outputs for P=Ksh100, 000, i=0.05, and n=10 years. [5marks]

c) Below is a Lagrange interpolating polynomial function . Explain the purpose of line 14, 15 and 18. [3marks]

```
1  function yint=Lagrange(x,y,xx)
2  % Lagrange: Lagrange interpolating polynomial
3  % yint = Lagrange(x,y,xx): Uses an (n - 1)-order
4  % Lagrange interpolating polynomial based on n data points
5  % to determine a value of the dependent variable (yint) at
6  % a given value of the independent variable, xx.
7  % input:
8  % x = independent variable
9  % y = dependent variable
10 % xx = value of independent variable at which the
11 % interpolation is calculated
12 % output:
13 % yint = interpolated value of dependent variable
14 n = length(x);
15 if length(y)~=n, error('x and y must be same length'); end
16 s = 0;
17 for i = 1:n
18     product = y(i);
19     for j = 1:n
20         if i ~= j
21             product=product*(xx-x(j))/(x(i)-x(j));
22         end
23     end
24     s = s+product;
25 end
26 yint = s;
```

d) Given the x and y values shown in the table below, what is the output of line 14 in the Langrange code above. Write down all the outputs of line 21 for $i = 1$ [5marks]

x	1	2	2.5	3	4	5
y	0	5	6.5	7	3	1

e) How will you call the function in (c) above to calculate y(2.8)?[1mark]

QUESTION THREE [15 MARKS]

a) Given the equation

$$\begin{aligned} 10x_1 + 2x_2 - \ \ x_3 &= \ \ 27 \\ -3x_1 - 5x_2 + 2x_3 &= -61.5 \\ x_1 + \ \ x_2 + 6x_3 &= -21.5 \end{aligned}$$

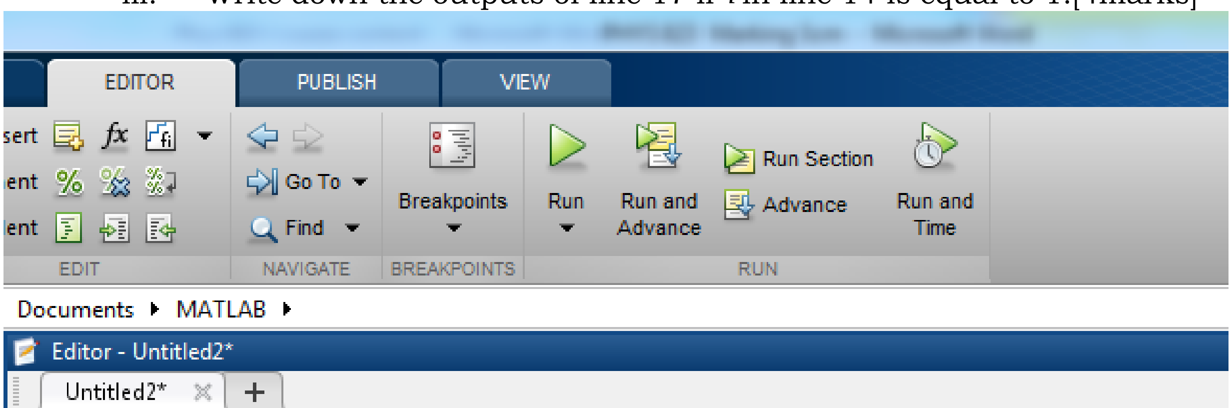
Solve by naive Gauss elimination show all steps of the computation. [3marks]

b) Below is a an unsaved function to implement the Naïve Gauss elimination in (a) above.

i. What is the correct file name of this function? [1mark]

ii. What will be the output of line 9 and 11 of the function when used to solve the problem in (a). [2marks]

iii. Write down the outputs of line 17 if i in line 14 is equal to 1.[4marks]




```
1  function x = GaussNaive_1(A,b)
2  % GaussNaive: naive Gauss elimination
3  % x = GaussNaive(A,b): Gauss elimination without pivoting.
4  % input:
5  % A = coefficient matrix
6  % b = right hand side solution vector in column form
7  % output:
8  % x = solution vector
9  [m,n] = size(A);
10 if m~=n, error('Matrix A must be square'); end
11 a = [A b];
12 % FORWARD ELIMINATION
13 nb = n+1;
14 for i = 1:n-1
15 for j = i+1:n
16 factor = a(j,i)/a(i,i);
17 a(j,i:nb) = a(j,i:nb)-factor*a(i,i:nb);
18 end
19 end
20 % BACK SUBSTITUTION
21 x = zeros(n,1);
22 x(n) = a(n,nb)/a(n,n);
23 for j = n-1:-1:1
24 x(j) = (a(j,nb)-a(j,j+1:n)*x(j+1:n))/a(j,j);
25 end
```

c) RungeKuttais one of thecommonly used numerical method for solving ODE due to its high degree of accuracy.It can be implemented through the steps below. The initial conditions are $x=x_0$, $y=y_0$ with x ranging from x_0-x_n and step size is h

1. Identify x_0, y_0 and h, and values of $x_1, x_2, x_3, \ldots$.
2. Evaluate $k_1 = f(x_n, y_n)$ starting with $n = 0$
3. Evaluate $k_2 = f\left(x_n + \frac{h}{2},\ y_n + \frac{h}{2}k_1\right)$
4. Evaluate $k_3 = f\left(x_n + \frac{h}{2},\ y_n + \frac{h}{2}k_2\right)$
5. Evaluate $k_4 = f\left(x_n + h,\ y_n + hk_3\right)$
6. Use the values determined from steps 2 to 5 to evaluate:

   $$y_{n+1} = y_n + \frac{h}{6}\{k_1 + 2k_2 + 2k_3 + k_4\}$$
7. Repeat steps 2 to 6 for $n = 1, 2, 3, \ldots$

Below is the beginning of the function that implements the above steps. Complete it. [5marks]

```
function [x,y] = rungekutta(dydx,xi,xf,yi,h)
% 4th-order Runge-Kutta integration.
% USAGE: [x,y] = rungekutt4(dydx,xi,xf,yi,h)
% INPUT:
% dydx = handle of function that specifies the
% 1st-order differential equation
% xi,yi = initial values.
% xf = final value of x.
% h = increment of x
% OUTPUT:
% x = x-values at which solution is computed.
% y = values of y corresponding to the x-values.
```

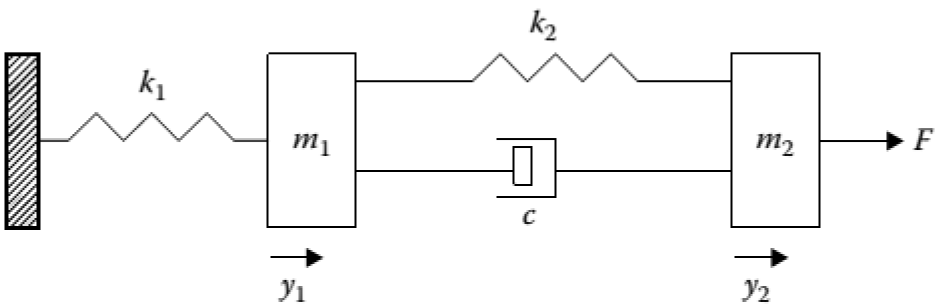
QUESTION FOUR [15 MARKS]

a) What is a system model? Why do we need it? [1marks]

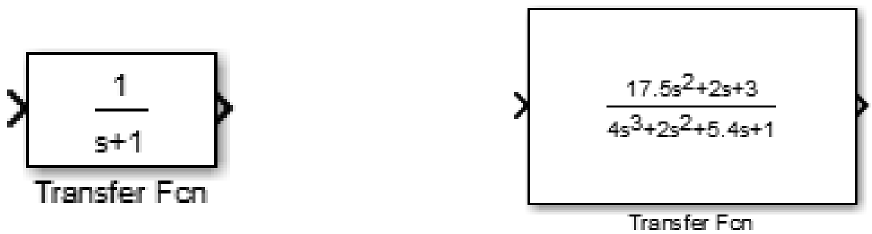
b) Explain three classifications of models [3marks

c) Name the steps involved in model development [2marks]

d) Figure below shows a mechanical system with two masses and two springs. Obtain the mathematical model of the system. [4marks]

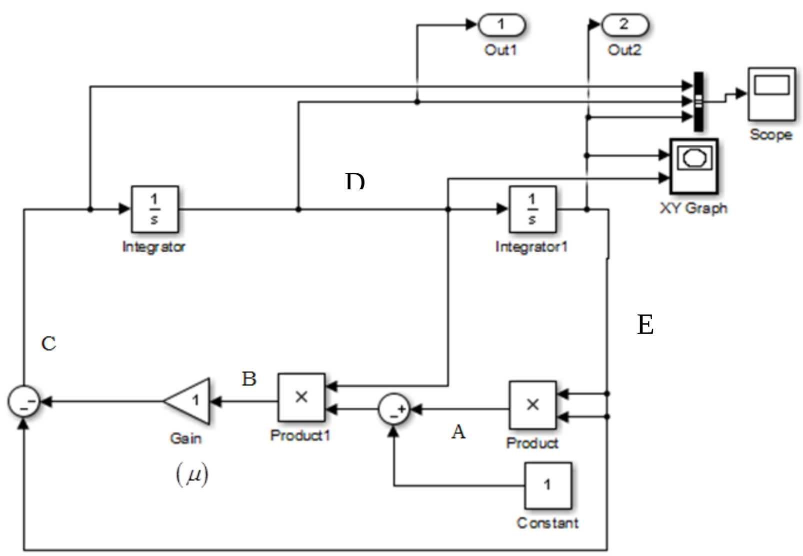


e) Figure (a) below shows a Simulink transfer block. Explain how the model can be modified to represent the transfer block of figure (b).
[2marks]



(a) (b)

f) Figure below is Simulink model to simulate a mathematical model.



i. State the outputs at pointsB,C, D and E [2marks]
ii. Write down the mathematical model equation represented by the Simulink model. [1 mark]