KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE (GEOPHYSICS AND MINEROLOGY)

FIRST SEMESTER 2021/2022
(FEBRUARY-JUNE, 2022)

PHRE215/BSMN 215: MATHEMATICAL METHODS

STREAM: Y2 S1 **TIME: 2 HOURS**

DAY: TUESDAY, 9:00 AM – 11:00 AM **DATE: 10/05/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer Question ONE (Compulsory) and any other TWO Questions.***

QUESTION ONE

a) Use Maclaurin's series to develop a power series approximation up to the term in x^7 of the function:

$$f(x) = ln\left(\frac{x+2}{2-x}\right)$$

(3 marks)

b) The gamma function is given by the integral

$$\Gamma(x) = \int_0^\infty (t^{x-1}\,e^{-t})dt, \;\; x > 0.$$

Show that $\Gamma(x+3) = (x+2)(x+1)x\Gamma(x)$ (4 marks)

c) Use De Movre's theorem to evaluate z^8 where $z = -2 + i\sqrt{7}$. (3 marks)

e) A d.c circuit comprises of three loops. Applying Kirchoff's laws to the closed loops gives the following equations for current flow:

$$3I_1 + 3I_2 - 4I_3 = 26$$
$$2I_1 - 5I_2 - 3I_3 = -54$$
$$-7I_1 + 2I_2 + 6I_3 = 26$$

Use Gaussian elimination method to solve for I_1, I_2 and I_2. (5 marks)

f) If $\vec{\boldsymbol{p}} = 2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{3k}$ and $\vec{\boldsymbol{q}} = \boldsymbol{4i} - 3\boldsymbol{j} + 2\boldsymbol{k}$, determine:

(i) $\vec{q}.\vec{p}$ **(2 marks)**

(ii) $\vec{q}x\vec{p}$ **(3 marks)**

g) Find the Laplace transform of the function $f(t) = t$.

(3 marks)

h) Given the partial differential equation, $f(x) = 4\sin(3x)\cos(2t)$ evaluate $\frac{\partial^2 f(x)}{\partial x^2}$ and $\frac{\partial f(x)}{\partial t}$. (4 marks)

i) Show that the function $f(x) = x - x^3$ in the interval $-\pi \leq x \leq \pi$ is an even function. (3 marks)

QUESTION TWO

a) Given the following function, find y'''.

$y(x) = \frac{1}{3}e^{-\frac{1}{2}x}$ (5 marks)

b) Find y''' if $y = \log(ax + b)$ (5 marks)

c) Given $z_1 = 2 + 4i$ and $z_2 = 3 - i$, determine $|z_1 + z_2|^2$ where z_1,z_2 are complex numbers. (3 marks)

d) Find the eigen values that satisfy the following matrix equation

$$\begin{vmatrix} (1-\lambda) & 4 \\ -1 & (2-\lambda) \end{vmatrix} = 0$$

(3 marks)

e) Use integration by parts to evaluate the integral:

$$\int_1^2 x^2 (\ln x)\ dx$$

(4 marks)

QUESTION THREE

a) Find Legendre polynomials P_1, P_2 and P_3 using Rodrigues formalism.

(3 marks)

b) Given that z is a complex number, determine z^6 if

$$z = ln(2 + 5i)$$

(4 marks)

c) The forces in three members of a framework are $F_1, F_2,$ and F_3. They are related by the following simultaneous equations:

$$0.7F_1 + 0.8F_2 + 1.8F_3 = 5.6$$
$$0.2F_1 - 1.4F_2 + 1.6F_3 = 35.0$$
$$0.4F_1 - 2F_2 - 1.3F_3 = -5.6$$

Find the values of $F_1, F_2,$ and F_3 using determinants. (4 marks)

d) Resolve the following into partial fractions then evaluate the integral

$$\int_2^3 \frac{x^3 - 2x^2 - 4x - 4}{x^2 + x - 2} dx$$

(5 marks)

e) If $u(x, y) = (\frac{x}{y}) ln\, y$, Show that $\frac{\partial u}{\partial y} = x \frac{\partial^2 u}{\partial y \partial x}$, and evaluate $\frac{\partial^2 u}{\partial y^2}$ when $x = -2$ and $y = 2$ (4 marks)

QUESTION FOUR

a) Find the Fourier series representing the function, $f(x) = x + x^2, \ for$

$-\pi < x < \pi$. (5 marks)

b) Evaluate the triple integral $\iiint_1^2 4(x^3 - x^{-2})dx$ (5 marks)

c) The velocity v of point P on a body with angular velocity ω about a fixed axis is given by

$$\vec{\boldsymbol{v}} = \vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{r}}$$

Where r is point on vector P. Find $\vec{\boldsymbol{v}}$ given that at point P, $\vec{\boldsymbol{\omega}} = \mathbf{2}\boldsymbol{i} - \mathbf{5}\boldsymbol{j} + \mathbf{7}\boldsymbol{k}$ and $\vec{\boldsymbol{r}} = \boldsymbol{j} + \mathbf{3}\boldsymbol{k}$, where $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are unit vectors. (4 marks)

d) Given $\vec{\boldsymbol{p}} = 3\boldsymbol{i} + 2\boldsymbol{k}, \vec{\boldsymbol{q}} = 4\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ and $\vec{\boldsymbol{r}} = 3\boldsymbol{i} + 5\boldsymbol{j} - 4\boldsymbol{k}$, determine:

(i) $-\vec{p} + 2|\vec{r}|$ **(2 marks)**

(ii) $(\vec{q} - 2\vec{p}).\vec{r}$ **(2 marks)**

(iii) $(\vec{r} + \vec{p}) \times \vec{q}$ **(2 marks)**

QUESTION FIVE

a) Find the constant term a_0 and a_1 of the Fourier series for the function

$f(x) = x$ in the interval $-\pi < x < \pi$. (3 marks)

b) Using the method of integration by parts evaluate the integral:

$$\int (x\cos x)\, dx$$

(3 marks)

c) Given $A = \begin{pmatrix} 3 & 4 & 0 \\ -2 & 6 & -3 \\ 7 & -4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -5 \\ 5 & -6 \\ -1 & -7 \end{pmatrix}$, find $A \times B$ (4 marks)

d) Find the eigen values λ that satisfy the following equation

$$\begin{vmatrix} (5-\lambda) & 7 & -5 \\ 0 & 4-\lambda & -1 \\ 2 & 8 & -3-\lambda \end{vmatrix} = 0$$

(5 marks)

e) The tensions $T_1, T_2,$ and T_3 in a simple framework are given by the following equations:

$$5F_1 + 5F_2 + 5F_3 = 7.0$$
$$F_1 + 2F_2 + 4F_3 = 2.4$$
$$4F_1 + 2F_2 = 4.0$$

Use Gaussian elimination to find the values of $T_1, T_2,$ and T_3. (5 marks)