KISII UNIVERSITY

UNIVERSITY EXAMINATIONS
FOURTH YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

SECOND SEMESTER 2021/2022
(FEBRUARY – JUNE, 2022)

PHYS 411: QUANTUM MECHANICS II

STREAM: Y4 S1 | **TIME: 2 HOURS**

DAY: TUESDAY, 3:00 PM – 5:00 PM | **DATE: 10/05/2022**

INSTRUCTIONS

1. ***Do not write anything on this question paper.***
2. ***Answer Question ONE (Compulsory) and any other TWO Questions.***

QUESTION ONE

a) By considering anistropicoscillayor in a three dimentional potential, write the eigen energies corresponding to the potential. [2 marks]

b) Define the term Hermitian operator and using the wave function ($\boldsymbol{\psi}$) gives its mathematical expression. [2 marks]

c) Taking
$\hat{J}_{\pm} = \hat{J}_x \pm i\hat{J}_y$

Show that:

$[\hat{J}_+, \hat{J}_-] = 2\hbar\hat{J}_z$ [4 marks]

d) Derive the following commutation relationship: [4 marks]

i. $[L_+, L_Z] = -\hbar L_+$

ii. $[L_-, L_Z] = -\hbar L_-$

e) Given that the electrons total momentum vector is given by; [4 marks]

$J = LxS$ and that $L \times L = i\hbar L$ and $S \times S = i\hbar s$

Find $J \times J$

f) What problem does stationary perturbation theory try to solve in quantum mechanics. [2 marks]

g)

h) Differentiate between degenerate perturbation theory and non-degenerate perturbation theory. (4 marks)

i) Prove that;

$J^2 = L^2 + S^2 + 2L.s$ [4 marks]

j) Write L_x, L_y *and* L_z in the spherical polar coordinates using r, θ *and* φ. [3mks]

QUESTION TWO

a) show that for a quantum mechanical oscillator the harmonic oscillator eigen function is given by: (10 marks)

$$E_n = \left(n + \frac{1}{2}\right)\hbar\boldsymbol{\omega}$$

b) systematically show that the time independent schrodinger equation in spherical coordinates takes the form: (10 marks)

$$\left[-\frac{\hbar^2}{2m}\frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{2mr^2}L^2 + V_{(r)}\right]\Psi(\mathrm{r}) = \mathrm{E}\Psi(\mathrm{r})$$

QUESTION THREE

a) Systematically show that in the degenerate perturbation theory, the first order correction in the energy is given by; [10 marks]

$$\sum_{\alpha=1}^{f}(\widehat{H}_{P,\beta\alpha}\text{-}E_n^{(1)}\delta_{\alpha,\beta})\mathrm{a}\alpha\text{=}0$$

Where $\beta = 1,2,3, \ldots \ldots \ldots$.f

b) In the first oder correction in energy E_n^1 show that; [10 marks]

$$I\psi_n >= I\varphi_n >= \sum_{m\neq n} \frac{\langle \varphi_m | \widehat{H}_p | \varphi_n \rangle}{E_n^{(0)} - E_m^{(0)}}$$

QUESTION FOUR

a) What are the goals of time dependent perturbation theory. [3 marks]

b) Show that; [6 marks]

$$\sum_n i\hbar \frac{\partial c_n}{\partial t} - c_n(t)V(t)e^{\frac{-iE_n t}{\hbar}} In >= 0$$

c) What are the main differences between time independence perturbation theory and time dependent perturbation theory. [4marks]

d) For time dependent perturbation theory, show that; [7mks]

$i\hbar \frac{dc_n(t)}{dt} = \sum_m H_{nm}(t) \exp(\boldsymbol{i\omega_{nm}t})\, \boldsymbol{C_m(t)}$

QUESTON FIVE

a) A particle of charge q and mass M, which is moving in one dimension. Harmonic potential of frequency ω is subjected to a weak electric field E in the x-direction.
   i) Find the expression for the energy. [5 marks]

   ii) Calculate the energy to the first non-zero correction and compare it to the exact result obtained in (a) [10 marks]

b) By clearly showing each step, prove that the following equation are correct;

   $[L^2, L_x] = [L^2, L_y] = [L^2, L_z] = 0$

   [5mks]