

  
**KISII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**

**SPECIAL EXAMINATION**  
**FIRST YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF**  
**BACHELOR OF SCIENCE RENEWABLE ENERGY/GEOPHYSICS &**  
**MINERALOGY**  
**SECOND SEMESTER 2021/2022**  
**(JULY, 2022)**

**BSMN124/PHRE 124: CALCULUS 1**

**STREAM: Y1 S2**

**TIME: 2 HOURS**

**DAY: THURSDAY, 8:30AM – 10:00 AM**

**DATE: 21/07/2022**

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**INSTRUCTIONS:**

- 1. Do not write anything on this question paper.**
- 2. Answer Question ONE (Compulsory) and any other TWO questions.**

**QUESTION ONE**

- a) What is a rational number? (2 marks)
- b) If  $A = \{a, b, e, f, h, i\}$   
 $B = \{a, c, e, g\}$  And  $C = \{d, g, i\}$   
Find (4 marks)
- (i)  $A \cap B$
- (ii)  $A \cap B \cap C$
- (iii)  $B \cup C$
- c) Use definition of derivative to find the derivative of  
$$v(t) = \frac{t+1}{t+4}$$
- d) Estimate the value of (3marks)

$$\lim_{x \rightarrow \infty} (2x^4 - x^2 - 8x)$$

e) Given the function

$$f(x) = \begin{cases} 7-4x & x < 1 \\ x^2+2 & x \geq 1 \end{cases}$$

Evaluate the following limits, if they exist

(i)  $\lim_{x \rightarrow -6} f(x)$  (2 marks)

(ii)  $\lim_{x \rightarrow 1} f(x)$  (2 marks)

(f) Determine where the given function is discontinuous (3 marks)

$$Y(x) = \frac{x}{7 - e^{2x+3}}$$

g) The position of an object at any time  $t$  is given by

$$s(t) = \frac{t+1}{t+4}$$

(i) Determine the velocity of the object at any time (3 marks)

(ii) Does the object ever stop moving (1 mark)

h) find the derivative of

i)  $f(x) = \frac{\sqrt{x} + 2x}{7x - 4x^2}$  (2 marks)

ii)  $f(x) = \frac{1+5t}{\ln t}$  (3 marks)

## QUESTION TWO

a), Differentiate the given function (4 marks)

$$h(u) = \tan(4+10u)$$

b) Determine all the numbers which satisfy the mean value theorem and interval for the given function and interval.

i)  $h(z) = 4z^3 - 8z^2 + 7z - 2$  [2, 5] (3 marks)

ii)  $A(t) = 8t + e^{-3t}$  [-2, 3] (3 marks)

c) Show that  $f(x) = x^3 - 7x^2 + 25x + 8$  has real root (3 marks)

d) Use L' Hospital's rule to evaluate each of the following limits

i)  $\lim_{w \rightarrow 4} \frac{\sin(\pi w)}{w^2 - 16}$  (4 marks)

ii)  $\lim_{z \rightarrow 0} \frac{\sin(2z) + 7z^2 - 2z}{z^2(z+1)^2}$  (3 marks)

### QUESTION THREE

a) Define a sequence of a real number (3marks)

b) Let  $r$  be a real number such that  $|r| < 1$ , prove that

$$\lim_{n \rightarrow \infty} r^n = 0 \text{ (5marks)}$$

c) Prove that

$$\lim_{n \rightarrow \infty} \frac{n^2 - n - 1}{2n^2 - 1} = \frac{1}{2} \quad \text{(5marks)}$$

d) Show that a sequence  $a_n$  converges to  $a$  if and only if, for every open interval  $I$  containing  $a$  there exists  $N \in \mathbb{N}$  such that  $a_n \in I$  for all  $n \geq N$  (7 marks)

### QUESTION FOUR

a) Find the Taylor series for the following functions

i)  $f(x) = \ln(3+4x)$  About  $x=0$  (4 marks)

ii)  $f(x) = 7x^2 - 6x + 1$  About  $x=2$  (4 marks)

b) Evaluate the limit of

i)  $\lim_{x \rightarrow \infty} f(x) = \ell^{8+2x-x^3}$  (3 marks)

ii)  $\lim_{t \rightarrow \infty} \tan^{-1}\left(\frac{4+7t}{2-t}\right)$  (3 marks)

c) Use chain rule to differentiate

i)  $f(x) = (6x^2 + 7x)^4$  (3 marks)

ii)  $f(t) = \ell^{1-\cos(t)}$  (3 marks)

### QUESTION FIVE

- a. Find the domain and range of  
 $Y(t) = 3t^2 - 2t + 1$  (3 marks)
- b. Given that  $7x \leq f(x) \leq 3x^2 + 2$  for all  $x$  determine the value of  
 $\lim_{x \rightarrow 2} f(x)$  (4 marks)
- c. Evaluate  $\lim_{x \rightarrow \infty} \frac{11+8x}{x^3+7x}$  [3mks]
- d. Determine where, if anywhere, the function  
 $g(x) = x^3 - 2x^2 + x - 1$  Stops changing. (3 marks)
- e. Find the tangent line to  $f(x) = 4\sqrt{2x} - 6e^{2-x}$  at  $x=2$  (3 marks)
- f. Find the  $n$ th derivative of  
 $\sin 5x \cdot \sin 3x$  [4mks]