KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SPECIAL EXAMINATION

THIRD YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE RENEWABLE ENERGY /GEOPHYSICS

SECOND SEMESTER 2021/2022

(JULY, 2022)

PHYS 322: QUANTUM MECHANICS 1

STREAM: Y3 S2 **TIME: 2 HOURS**

DAY: FRIDAY, 8:00 AM – 10:00 AM **DATE: 29/07/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer Question ONE (Compulsory) and any other TWO questions.***

QUESTION ONE [30 marks]

a) For the wave function $\boldsymbol{\psi(x) = A(ax - x^2)}$ for $0 \leq x \leq a$, where A is a constant normalise the wave function [6 marks]

b) Consider the wave function $\boldsymbol{\psi(x) = \sqrt{\frac{1}{2}}}$ **,** using the relation $\boldsymbol{\phi(p) = \sqrt{\frac{1}{2\pi\hbar}} \int_{-a}^{a} \psi(x)\, e^{\frac{-ipx}{\hbar}} dx}$ **,** obtain, Momentum space wave function [6 marks]

c) Heisenberg's uncertainty principle relates the position and the momentum of a particle clearly state this principle and outline its physical significance [4 Marks]

d) For a normalised ground state wave function of an oscillator, find the probability that the particle is found between $\boldsymbol{a \leq x \leq 2a}$[6 marks]

e) Information about a state make a wave function valid of a quantum system is contained in the wave functions outline the four properties of a valid wave function? [5 marks]

f) Suppose(x,t) = A(x − x^3)exp(−iEt/h) Find V(x) such that the Schrödinger equation is satisfied. [3 marks]

QUESTION TWO

a) For a particle of mass m trapped in 1 dimensional potential defined by

$$V(x) = \begin{cases} 0 & 0 \leq x \leq a \\ \infty & \text{otherwise} \end{cases}$$

Solve the Schrodinger equation and obtained the general expression for the ground state wave function and energy [10 marks]

2) Using the wave function obtained in a) compute the expectation values $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, $\langle p^2 \rangle$, [10 marks]

QUESTION THREE

A beam of particles coming from **x = −∞** meets a potential barrier described by **V(x) = V**, where **V** is a positive constant, at **x = 0**.

Suppose particles have energy **0 < E < V**.

a) Sketch this potential [3 Marks]
b) Find the wave function for **x < 0** and for **x > 0**, [5 Marks]
c) Describe the transmission and reflection coefficients for this potential barrier [8 Marks]
d) Describe the behavior of the wave function behave on both sides of the barrier? Indicate using a sketch. Comment on your results. [4 Marks]

QUESTION FOUR

The operators for $\boldsymbol{J^2}$and $\boldsymbol{J_z}$are diagonal in the basis given by their jointeigenstates

a) Find the matrix representation of the operators $\boldsymbol{J^2}$ and $\boldsymbol{J_z}$for the case where $\boldsymbol{j = 1}$

a) Obtain the joint eigenstates of $\boldsymbol{J^2}$ and $\boldsymbol{J_z}$
b) Use the matrices to calculates $[\mathbf{J_y}, \mathbf{J_z}]$

QUESTION FIVE

Consider a delta function perturbation $\boldsymbol{H' = \alpha\delta(x - \frac{a}{2})}$at the centre of an infinite square, where $\boldsymbol{\alpha}$is a constant

a) Find the first-order correction to the allowed energies. Explain why energies are not perturbed for even n

b) Find the first three nonzero terms in the expansion of the correction to theground state wave function .

$$\psi_n^1 = \sum_{m \neq n} \frac{< \psi_m^o | H' | \psi_n^o >}{E_n^o - E_m^o} \psi_m^o$$