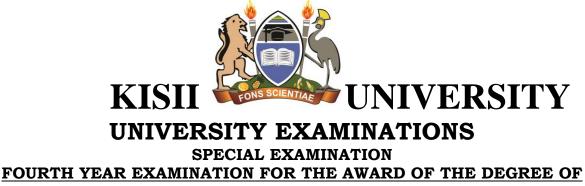
PHYS 411



BACHELOR OF EDUCATION (SCIENCE) FIRST SEMESTER 2021/2022

<u>(JULY, 2022)</u>

PHYS 411: QUAMTUM MECHANICS

STREAM: Y4 S1

TIME: 2 HOURS

DATE: 28/07/2022

DAY: THURSDAY, 8:00 AM - 10:00 AM

INSTRUCTIONS:

- 1. Do not write anything on this question paper.
- 2. Answer Question ONE (Compulsory) and any other TWO questions.

QUESTION ONE

- a) What are the Main differences between spin wave function and the regular wave function? [2 marks]
- b) Define the term Hermitian operator and using the wave function (ψ) gives its mathematical expression. [2 marks]
- c) Show that: [4mks]

$$L_{+}L_{-} = L^{2} - L_{Z}^{2} + \hbar L_{Z}$$

- d) Derive the following commutation relationship: [4 marks]
 - i. $[L_+, L_Z] = -\hbar L_+$
 - ii. $[L_-, L_Z] = -\hbar L_-$
- e) Given that the electrons total momentum vector is given by; [4 marks] J = LxS and that $L \times L = i\hbar L$ and $S \times S = i\hbar s$ Find $J \times J$
- f) What problem does stationary perturbation theory try to solve in quantum mechanics. [2 marks]

- g) Differentiate between degenerate perturbation theory and non-degenerate perturbation theory.
 (4 marks)
- h) Prove that; $J^2 = L^2 + S^2 + 2L.s$ [4 marks]
- i) Write L_x, L_y and L_z in the spherical polar coordinates using r, θ and φ . [3 marks]

QUESTION TWO

a) Systematically show that in the degenerate perturbation theory, the first order correction in the energy is given by; [10 marks]

 $\sum_{\alpha=1}^{f} (\widehat{H}_{P,\beta\alpha} - E_n^{(1)} \delta_{\alpha,\beta}) a\alpha = 0$

Where $\beta = 1,2,3, \dots \dots \dots f$

b) In the first order correction in energy E_n^1 show that; [10 marks]

$$I\psi_n >= I\varphi_n >= \sum_{m \neq n} \frac{\langle \varphi_m | \hat{H}_p | \varphi_n \rangle}{E_n^{(0)} - E_m^{(0)}}$$

QUESTION THREE

- a) Use the Wentzel-Kramers-Brilliuon method to estimate the level of one dimensional harmonic oscillator. [7 marks]
- b) Use the Wentzel-Kramers-Brilliuon approximation to calculate the levels of spinless particle of mass M moving in a one dimensional box with walls at x=0 and x=L.
- c) Use Wentzel-Kramers-Brilliuon approximation to estimate the transmission coefficient of a particle of mass M and energy E moving in the following potential barriers. [7 marks]

$$V_{(x)} = \begin{cases} 0, & x < 0 \\ V_o - \lambda, x & x > 0 \end{cases}$$

QUESTION FOUR

b) Show that;

- a) What are the goals of time dependent perturbation theory. [3 marks]
 - [6 marks]
 - $\sum_{n} i\hbar \frac{\partial c_{n}}{\partial t} c_{n}(t)V(t)e^{\frac{-iE_{n}t}{\hbar}}In \ge 0$

- c) What are the main differences between time independence perturbation theory and time dependent perturbation theory. [4 marks]
- d) For time dependent perturbation theory, show that; [7 marks] $i\hbar \frac{dc_n(t)}{dt} = \sum_m H_{nm}(t) \exp(i\omega_{nm}t) C_m(t)$

QUESTON FIVE

- a) A particle of charge q and mass M, which is moving in one dimension. Harmonic potential of frequency ω is subjected to a weak electric field E in the x-direction.
 - i) Find the expression for the energy. [5 marks]
 - ii) Calculate the energy to the first non-zero correction and compare it to the exact result obtained in (a) [10 marks]
- **b)** By clearly showing each step, prove that the following equation are correct;

 $[L^2, L_x] = 0$

[5 marks]