KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF MASTERS OF APPLIED STATISTICS
SECOND SEMESTER 2022/2023
[JANUARY – APRIL, 2023]

MATH 884: MULTIVARIATE ANALYSIS II

STREAM: Y1 S2 | **TIME: 2 HOURS**

DAY: MONDAY, 9:00-12:00 P.M | **DATE: 20/03/2023**

INSTRUCTIONS

1. ***Do not write anything on this question paper.***
2. ***Answer question ONE (Compulsory) and any other TWO questions.***

QUESTION ONE (30 MARKS)

a) What is mean by Discriminant analysis in multivariate methods. [2marks]

b) Prove that if X_1 and Z are independent then $Cov(X_1, Z) = 0$ [5marks]

c) Given that $X \sim N_p(\mu, \varepsilon)$ with μ known and $\sum = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 7 \end{bmatrix}$, determine partial correlation coefficient between X_1, X_2 given X_3

d i) Explain correlation matrix (ρ) [1marks]

ii) Show that the correlation matrix (ρ) is symmetric [3marks]

e) Given that $\sum = E[X-\mu][X-\mu]^T$, show that $Cov(X) = \begin{Bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1p} \\ \delta_{21} & \delta_{22} & & \delta_{2p} \\ & \vdots & & \\ \delta_{p1} & \delta_{p2} & & \delta_{pp} \end{Bmatrix}$ (5marks)

f) Suppose X is a random Px1 vector with $Var(X) = \delta_{ij}$,

i) obtain the correlation coefficient between X_i and X_j (5marks)

ii) Show that $\rho^T = \rho$ (5marks)

g) Differentiate between discriminant analysis and Canonical analysis giving example. (4marks)

QUESTION TWO 20MARKS

Given the matrix of dispersion $\Sigma = \begin{Bmatrix} -1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{Bmatrix}$ which of the following random variables are independent?

a) X_1 *and* X_2
b) X_2 *and* X_3
c) X_1X_2 *and* X_3
d) X_3X_2 *and* X_1
e) $\frac{X_1+X_2}{2}$ *and* X_3
f) X_1 *and* $X_2 - X_3$ (20marks)

QUESTION THREE 20 MARKS

a) Explain what is meant by principal component analysis giving example where it is applicable in real life situation (4marks)

b) Determine the first principal component and the total variance explained by the component and correlation coefficient between the first principal component of the first variable given that

$$s = \begin{Bmatrix} 1 & 0 & 0.5 \\ 0 & 4 & 0 \\ 0.5 & 0 & 9 \end{Bmatrix}$$ (16marks)

QUESTION FOUR 20 MARKS

a) Discuss canonical correlation analysis (5marks)
b) Explain steps in determining measures of association between two random vectors x and y. (5marks)
c) i) Explain the meaning of multiple correlation coefficient (2marks)

ii) Given that $X \sim N(\mu, \Sigma)$ with $\mu = \begin{pmatrix} 10 \\ 6 \\ 12 \end{pmatrix}$ *and* $\Sigma = \begin{Bmatrix} 16 & 4 & 6 \\ 4 & 4 & 0 \\ 6 & 0 & 9 \end{Bmatrix}$

iii) Determine the distribution of $Y = \begin{Bmatrix} X_3 - X_1 \\ X_1 + 2X_2 - X_3 \end{Bmatrix}$

(8marks)

QUESTION FIVE (20 MARKS)

a i) Find the principal component and the total variance explained by the first principal and the first variable given that $S = \begin{bmatrix} 1 & 0 & 0.6 \\ 0 & 4 & 0 \\ 0.6 & 0 & 9 \end{bmatrix}$

[13marks]

ii) Determine the correlation between the first principal component and the variable X_k [2marks]

b) Suppose $U = a'X$ and $V = b'X$, show that $COV(U,V) = a' \sum b$

[5marks]