KISII UNIVERSITY

UNIVERSITY EXAMINATION 2022/ 2023

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE WITH INFORMATION TECHNOLOGY

MATH 102: DISCRETE MATHEMATICS

Instruction:

- Attempt **Question One** (Compulsory) and any other **two** questions.

QUESTION ONE (Compulsory) [30 Marks]

(a) Differentiate between:

    (i) *tautology* and *contradiction*. [2 Marks]

    (ii) *Universal quantifier* and *Existential quantifier*. [2 Marks]

(b) Provide a combinatorial proof of the identity: [4 Marks]

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}.$$

(c) Consider the statement, "Let x be an integer. If x^2 is even, then x is even." Prove the contrapositive. [3 Marks]

(d)   (i) State the *pigeonhole principle*. [1 Mark]

    (ii) What is the minimum number of students required in a discrete mathematics class to be sure that atleast six will receive the same grade, if there are five possible grades A, B, C, D and F. [3 Marks]

(e) What is the cardinality of the power set of $\{\emptyset\}$? [2 Marks]

(f)   (i) Determine whether $f(x) = x^2$ is one-to-one from a set of integers to a set of integers. [2 Marks]

    (ii) Use mathematical induction to prove that for all natural numbers n, the expression $4^n - 1$ is divisible by 3 [4 Marks]

(g)   (i) Find the greatest common divisor of 414 and 662 using the Euclidean algorithm. [3 Marks]

    (ii) What is the decimal expansion of the integer whose binary representation is $(101011111)_2$. [3 Marks]

(h) State the *principle of inclusion-exclusion*. [1 Mark]

QUESTION TWO [20 Marks]

(a) Using induction prove that $2^n \le n!$ for all integers with $n \ge 5$. [4 Marks]

(b) Show that $\sqrt{2}$ is irrational. [4 Marks]

(c) Let $f : Z \to Z$ be such that $f(x) = x + 1$. Is f invertible? If so, what is its inverse?

[2 Marks]

(d) Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g? and the composition of g and f. [4 Marks]

(e)   (i) State *Binomial Theorem*. [2 Marks]

    (ii) What is the coefficient of $x^{12}y^{13}$ in the expression $(2x - 3y)^{25}$ [4 Marks]

QUESTION THREE [20 Marks]

(a) If $A = \{a, b, c\}$ and $B = \{d, e\}$, find:

    (i) $\mathcal{P}(A)$, the power set of A. [2 Marks]

    (ii) $A \times B$. [2 Marks]

(b)   (i) Define *equal sets*. [1 Marks]

    (ii) Given $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$ and $B = \{3, 4, 5\}$. Show that:
$(A \cup B)^c = A^c \cap B^c$ [3 Marks]

    iii Using set builder notation, show that $(A \cap B)^c = A^c \cup B^c$. [3 Marks]

(c) Use a truth table to show that the statements $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.
[4 Marks]

(d) By use of correct logic gates, draw a circuit that gives $(p \wedge \neg q) \vee \neg r$. [5 Marks]

QUESTION FOUR [20 Marks]

(a) How many different ways are there to seat four people around a circular table where two seatings are considered the same when each person has the same left neighbour and same right neighbour. [5 Marks]

(b) Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department? [3 Marks]

(c) Find 2-permutation of the set $S = \{a, b, c\}$. Hence, list all the permutations.

[4 Marks]

(d) How many bit strings of length eight either start with a 1 or end with 00?

[4 Marks]

(e) Use a combinatorial reasoning, and not binomial theorem, to find the expansion of $(x + y)^3$.
[4 Marks]

QUESTION FIVE [20 Marks]

(a) (i) Show that 101 is prime. [2 Marks]

(ii) Find the prime factorization of 7007. [3 Marks]

(b) Use the Euclidean Algorithm to show that $gcd(252, 198) = 18$ is a linear combination of 252 and 198. [5 Marks]

(c) Find the inverse of 3 modulo 7. [5 Marks]

(d) Find all solutions to the linear congruence

$$3x \equiv 4 \ (\text{mod } 7).$$

[5 Marks]

Course Outline

- Formal logic: Propositional Logic, Predicate Logic

- Sets: Universes, Venn diagrams, Subsets, Set operations, Cardinality, Power sets, Partitions of sets, Cartesian products of sets

- Proof techniques: Direct proofs, Proof by counterexample, Proof by contradiction, Proof by induction, Contrapositive, Combinatorial Proofs, Vacuous Proofs, Trivial Proofs

- Basic counting techniques: Principles of counting, Permutations, Combinations, Binomial coefficients, Binomial and multinomial theorems, Pigeon-hole principle

- Elementary number theory: Positional number systems, Prime numbers, Factorization, Euclidean algorithm, Chinese Remainder Theorem, Residue number systems, Modular arithmetic