**UNIVERSITY EXAMINATIONS**

**2022 -2023 ACADEMIC YEAR SEMESTER TWO,**

**JANUARY-APRIL 2023**

**COURSE CODE: MATH 111**

**COURSE TITLE: CALCULUS 1**

**INSTRUCTIONS TO CANDIDATES**

**Answer question one (compulsory) and any other two questions**

**Time: 2 hours**

## QUESTION ONE (30 MARKS)

(a) (i)State three conditions for a function $f(x)$ to be continuous at the point $x = a$ (3mks)

(ii) Determine the continuity of the function $f(x) = f(x) = \begin{cases} x^2 + 2x, & x < -2 \\ x^3 - 4x, & x \geq -2 \end{cases}$ at $x = -2$ (6mks)

(b) (i) Determine the equations of both the tangent line and normal line to the curve $y = 3x^2 - x + 1$ at the point where $x = 1$ (4mks)

(ii) Differentiate $y^3 + \sin y = 0$ with respect to x. (2mks)

(c) Evaluate the following:

(i) $\lim_{x\to 2} \frac{x-2}{4-x^2}$ (2mks)

(ii) $\lim_{x\to\infty} \left(\frac{x^2+2x-3}{x^2-x}\right)$ (3mks)

(d) If $x = \cos t$ and $y = 1 - \sin 2t$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ (4mks)

(e) Determine the continuity of $f(x) = \frac{1}{x}$ over the interval $(0, \infty)$ (3mks)

(f) Find $\frac{dy}{dx}$ if $y = e^{3x}(\sin 2x)$ (3mks)

## QUESTION TWO (20 MARKS)

(a) At what point does $y = 3x^2 - 12x^2 + 45x - 55$ have an horizontal tangent (3mks)

(b) Differentiate the implicit function; $x^2 + 2y^3$ (3mks)

(c) Use the appropriate method of differentiation to differentiate the following

(i) $\frac{2\cos x}{\sin x}$ (4mks)

(ii) $y = \frac{x^2+5x-4}{2x+1}$ (3mks)

(iii) $y = \cos(6x^3)$ (3mks)

(d) Verify the mean value theorem for $f(x) = x^3 - 3x^2 - 10x + 20$ on the interval $[-1,5]$

(4mks)

**QUESTION THREE (20 MARKS)**

(a) Use Newton's method to find $\sqrt{17}$ (3mks)

(b) Given $5y^2 - 3x^2 - x + y = 0$, find $\frac{dy}{dx}$. Hence find the equation of the tangent and equation of the normal to the curve of $5y^2 - 3x^2 - x + y = 0$ at the point $(1, -1)$. (6mks)

(c) Differentiate the implicit function:

$x^3 + 2x^2 y^2 + x^2 = 4$ (4mks)

(d) Differentiate the following functions;

(i) $x^5 \cos x$ (ii) $\sin x^2$ (5mks)

(e) Differentiate $y^3 + \sin y = 0$ with respect to x. (2mks)

**QUESTION FOUR (20 MARKS)**

(a) Find the equation of the tangent to $y = (x+3)^{\frac{2}{3}}$ at the point (5,4) (4mks)

(b) Differentiate the following using appropriate methods.

(i) $(x^3 - 2)(\sin x + \cos x)$ (4mks)

(ii) $\frac{\tan x + 1}{\cos x}$ (5mks)

(iii) $x^2 + y^2 = \sqrt{7}$ (3mks)

(c) Find the dimensions of the rectangle that has maximum area if its perimeter is $20m$. Comment on the outcome. (4mks)

## QUESTION FIVE (20 MARKS)

(a) Define a parametric function. Hence find $\frac{dy}{dx}$ given y =$t^2 + 3$ and $x$ =$t^2 + 5t + 9$

(3mks)

(b) Given that $P = 3q^4 - 4q^2 + 3$; $find$: $\frac{d^3p}{dq^3}$ (3mks)

(c) (i) At what point does the tangent to the function $y = x^3 + 2x^2$ have a slope of zero. (4mks)

(ii) Differentiate; $y^3 + \sin x = 3$ (4mks)

(iii) Differentiate:$f(x) = e^{sin\,x}$ (2mks)

(d) Let $f(x) = x^3 + 1$. Show that $f$ satisfies the hypothesis of Mean value theorem on the interval $[1,2]$ and find all values of $c$ in this interval whose existence is guaranteed by the theorem. (4mks)

(e) Find from the first principles, the derivative of the function $f(x) = sinx$ (4mks)