KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED STATISTICS
SECOND SEMESTER 2022/2023
[JANUARY-APRIL, 2023]

MATH 116: MATRIX ALGEBRA

STREAM: Y1S2   **TIME: 2 HOURS**

DAY: WEDNESDAY, 9:00 – 11:00 AM   **DATE: 05/04/2023**

INSTRUCTIONS
1. **Do not write anything on this question paper.**
2. *Answer question ONE and any other TWO questions.*

QUESTION ONE (COMPULSORY)   (30 MARKS)

a) Calculate the determinant of the following matrices:

i) $A = \begin{pmatrix} \frac{1}{7} & \frac{2}{3} \\ \frac{1}{3} & -\frac{2}{5} \end{pmatrix}$ ii) $A = \begin{bmatrix} 4 & -7 & -6 \\ 3 & -1 & 4 \\ -8 & 2 & -5 \end{bmatrix}$ (6marks)

b) Given the matrices $A = \begin{bmatrix} -1 & 2 & -1 \\ 1 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$, determine $(AB)^{-1}$

(7marks)

c) Use matrices to solve:

i) $3u + 2a = 13$
$-16u + 6a = -36$ (3marks)

ii) $x + 2y - 3z = 3$
$2x - y - z = 11$
$3x + 2y + z = -5$ (5marks)

d) Given the matrix $B = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$ and that $B^2 - 5B + kI = 0$, where k is a constant, determine the value of k.   (5marks)

e) Use Cramer's rule to solve:

$$5I_1 + 5I_2 + 5I_3 = 7$$
$$I_1 + 2I_2 + 4I_3 = 2.4$$
$$4I_1 + 2I_2 = 4$$

(4marks)

QUESTION TWO   (20MARKS)

a) Use Gauss-Elimination method to solve:
$$10x - 2y - 3z = 205$$
$$2x - 10y + 2z = -154$$
$$2x + y - 10z = -120$$
(6marks)

b) Find the eigenvalues of the matrix:
$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$
(7marks)

c) Use the method of determinants to solve the simultaneous equations.
$$14x - 8y = 4$$
$$-8x + 10y - 6z = 20$$
$$-6y + 104z = -28$$
(7marks)

QUESTION THREE   (20MARKS)

a) Find the characteristic polynomial of the matrix
$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$
Hence, find A^{-1}.
(7marks)

b) Solve the equation $\begin{vmatrix} 8 & -2x \\ 10 & 4x \end{vmatrix} = 8$
(3marks)

c) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$
(10marks)

QUESTION FOUR   (20MARKS)

Find the modal matrix P and the resulting diagonal matrix D of A, if:

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$
(20marks)

QUESTION FIVE   (20MARKS)

a) Reduce quadratic form to canonical form using orthogonal transformation.
$$3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_3x_1 - 2x_1x_2$$
(12marks)

b) The relationship between the displacement, s, velocity, v, and acceleration, a, of a piston is given by the equations:
$$6s + 4v - 4a = 64$$
$$8s + 6v + 6a = 8$$
$$-4s + 2v - 2a = 4$$
Use matrices to determine the values of s, v and a.
(8marks)