MAT 222-ALGEBRAIC STRUCTURES

Answer Question one and any other two questions

QUESTION ONE(COMPULSORY)-30Marks

1(a). Let A and B be two sets. Define the intersection of A and B and the set difference of B from A. (2marks)

b.) Let f be a mapping from a set X to a set Y. When is f is said to be *injective and surjective* (3marks)

c.) Let m, n $\in \mathbb{N}$. Define addition and multiplication of m and n and show their commutative ,associative and distributive properties. (5marks)

d.) Show that every non empty subset X of $\mathbb{N}$ has a least element. (3marks)

e.) Distinguish between group and a belian group (2marks)

f.) Let $*$ be a binary operation on a set S. Define identity elements for S (2marks)

g.) Let G be a non-empty set and $*$ a binary operation . When do we call $(G, *)$ a group and commutative? (3marks)

h.) Define *cyclic groups* giving relevant examples (4marks)

i.) Let F be a non empty set and let +,$*$ be binary operations on F. Define the field $(F, +, *)$ (4marks)

j.) Let $*$ binary operation on a set S. Let e, f $\in S$ be identity element for S with respect to $*$. Show that e=f. (2marks)

QUESTION TWO (20 Marks)

2.a) Show that every field is an integral domain and every finite integral domain is a field (10marks)

b.) If R is an integral domain and a, b c $\in R$ with a not zero and ab=ac then b=c (2marks)

c.) Define *integral domain* and *commutative ring with unity* giving detailed examples for each (8marks)

QUESTION THREE-(20 Marks)

3.a) What is a commutative rings (2arks)

b.) Discuss two examples of non-commutative rings (8marks)

b.) Discuss two examples of commutative rings (10marks)

QUESTION FOUR-(20 Marks)

4.a) What is a right coset . (2marks)

b.) Show that cosets are either identical or disjoint (6marks)

b.) Let S be a sub group of the group G and let a,b $\in G$ show that sa=sb if and only if $ab^{-1} \in$ S (6marks)

b.) Show that if |G|=p a prime , then G is cyclic. (6marks)

QUESTION FIVE-(20Marks)

5.a) State Lagrange theorem (2marks)

b.) Show that every subgroup of a cyclic group is cyclic (8marks)

c.) How many generators does a cyclic group of Order 400 have ? (4marks)

d. For each positive integers x ,how many elements of order x does a cyclic group of order 400 have? (6marks)