MAT 401-MEASURE THEORY

Answer Question one and any other two questions

QUESTION ONE(COMPULSORY)-30Marks

1(a). What is ment by a measure being **additive and countably additive**? (2marks)

b.) State the existence of Lebesgue theorem (3marks)

c.) Define Lebesgue outer measure (2marks)

d.) An open set comprises of an infinite disjoint open interval of the form I=$\{(1,6)(7,17)(23,43)\cdots\}$. Compute for any **n** the length of the first **n** open intervals hence specify for **n=5** (6marks)

e.) When is a subset E of $\mathbb{R}$ said to be Lebesgue measurable? (2marks)

f.) Show that if A and B are measurable subset of $\mathbb{R}$, then $A \cup B$ is also measurable (4marks)

g.) Prove that if $A \subseteq \mathbb{R}$ and $m*(A) = 0$,Then A is measurable (3marks)

h.) Check whether

$$\int_0^4 \frac{1}{\sqrt{x}}dx$$

is Lebesgue integrable (4marks)

i.) If $\{E_k\}$ is any sequence of measurable subsets of $\mathbb{R}$ then $\cup_{k\in\mathbb{N}}$ is measurable (4marks)

QUESTION TWO (20 Marks)

2.a) Show that every J in $\mathbb{R}$ is Lebesgue measurable (10marks)

b.) Prove that if J is any interval in $\mathbb{R}$, then m(J)=l(J) (10marks)

QUESTION THREE-(20 Marks)

3.a) Find the length of the set $\cup_{k=1}^{\infty} \{X : \frac{1}{k+1} \leq x < \frac{1}{k}\}$ (8marks)

b.) Show that if $S \subseteq T \subseteq \mathbb{R}$ then $m*(S) \leq m*(T)$, where m* is outer measure. (4marks)

c.) Compute the length open set $\cup_{k=1}^{\infty} \{X : \frac{1}{3^{k+1}} \leq x < \frac{1}{3^k}\}$ (8marks).

QUESTION FOUR-(20 Marks)

4.a) Use the concept of Riemann integrability to show for Lebesgues integrability in

$$\int_0^{\infty} \frac{dx}{(x+2)^3}$$

. (5marks)

b.) Show that f(x)=1 is both Riemann and Lebesque integrable on $E \in [a, b]$ (5marks)

b.) Show that

$$\int_{-7}^{2} \frac{dx}{(x-1)^{\frac{5}{3}}}$$

is not Lebesque integrable (10marks)

QUESTION FIVE-(20Marks)

5.a) State and prove *Fatou's Lemma* (8marks)

b.) Prove that for every countable subset of $\mathbb{R}$ is measurable and has a measure zero (5marks)

b.) State and prove Monotone convergence theorem (7marks)