KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE
SECOND SEMESTER 2022/2023
[JANUARY-APRIL, 2023]

PHYS 322: QUANTUM MECHANICS I

STREAM: Y3S2 **TIME: 2 HOURS**

DAY: THURSDAY, 9:00 – 11:00 AM **DATE: 20/04/2023**

INSTRUCTIONS

1. ***Do not write anything on this question paper.***
2. ***Answer question ONE and any other TWO questions.***

QUESTION ONE

(a) What property of radiation (light) was realized in both photoelectric effect and Compton effect? (2 marks)

(b) What are 'matter waves' ? (2 marks)

(c) State and explain de Broglie's hypothesis. (3 marks)

(d) Calculate the de Broglie's wavelength for:

(i) a photon of kinetic energy 70 MeV. (3 marks)

(ii) a $100g$ bullet moving at 900 m/s. (3 marks)

(e) Explain the concept of particle-wave duality as a complementarity.

(3 marks)

(f) State and explain Heinsberg's Uncertainty principle. (2 marks)

(g) Estimate the uncertainty in the position of:

(i) a neutron moving at $5.0 \times 10^{6}\ m/s$. (3 marks)

(ii) a 50.0 kg mass moving at 2.0 m/s. (3 marks)

(iii) comment on which case from (i) and (ii) above has meaningful uncertainty. (2 marks)

(h) (i) Explain what it means by 'Quantization rules'. (2 marks)

(ii) State Plank's quantization rule, and Bohr's quantization rules**.** (2 marks)

QUESTION TWO

(a) Derive the energy of the hydrogen atom using Bohr's formulas. (7 marks)

(b) Derive a relation that predicts the frequencies of the line spectra of hydrogen. (8 marks)

(c) A thermal neutron has a speed that corresponds to room temperature $T = 300\ K$. What is the wavelength of the thermal neuron? (5 marks).

QUESTION THREE

(a) Let two functions φ and ϕ be defined for $0 \leq x \leq \infty$. Explain why $\varphi(x) = x$ cannot be a wavefunction but $\phi(x) = e^{-x^2}$ would be a valid wavefunction. (4 marks).

(b) Consider a particle in a well with potential given by:

$$V(x) = \begin{cases} 0, & 0 \leq x \leq a \\ \infty, & otherwise \end{cases}$$

Show that $\psi(x,t) = A\, sin(kx) exp(iEt/\hbar)$ solves the Schrodinger equation provided that

$$E = \frac{\hbar^2 k^2}{2m}$$ (10 marks)

(c) Suppose $\psi(x,t) = A(x - x^2)exp(-iEt/\hbar)$. Find $V(x)$ such that the Schrodinger equation is satisfied. (6 marks)

QUESTION FOUR

(a) A particle of mass m is trapped in one dimensional box with a potential described by:

$$V(x) = \begin{cases} 0, & 0 \leq x \leq a \\ \infty, & otherwise \end{cases}$$

Solve the Schrodinger equation for this potential. (5 marks)

(b) Given the wavefunction

$$\psi(x,t) = A(x - x^2), \quad 0 \leq x \leq a$$

(i) Normalize the wavefunction (5 marks)

(ii) Find $\langle x \rangle, \langle x^2 \rangle$ *and* Δx (10 marks)

QUESTION FIVE

a) A particle of mass m in one dimensional box is found to be in the ground state:

$$\psi(x) = \sqrt{\frac{2}{a}}\ sin\left(\frac{\pi x}{a}\right).$$

Find $\Delta x \Delta p$ for this state. (10 marks)

(c) Suppose that a wavefunction is given by

$$\psi(x) = A(x^5 - ax^3)$$

Find Δx and Δp. (10 marks)