# KISII UNIVERSITY

## SCHOOL OF PURE AND APPLIED SCIENCES

## DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

## MSC APPLIED MATHEMATICS

## MAT 808: ORDINARY DIFFERENTIAL EQUATIONS II

<u>DATE: SEPT DEC 2022</u>                                      <u>FINAL EXAM</u>

*INSTRUCTIONS:  Answer question one and any other two questions*

**SECTION A (30 MARKS)**

1.

a. Define the following terms giving examples in each case;

   i.     First order linear differential equation           (5 marks)

   ii.    Ordinary differential equation                     (5 marks)

 b.  Show that $\frac{d^2y}{dx^2} - 9y = 0$ is linearly independent           (5 marks)

 c.  Use power series method to solve $(1 - x^2)y'' + 2y = 0$ at point

   $y(2) = 4, y'(2) = 5$                                    (5 marks)

 d. If $y1 = x^2$ is a solution of     $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$ find its general

   solution                                                 (5 marks)

 e. Solve $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 4y = 0$                      (5 marks)

**SECTION B (20 MARKS)**

2.

  a. Use the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = xe^x + 2x \qquad\qquad \text{(5 marks)}$$

  b. Solve the initial value problem by Laplace transform,

$$\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = 2, \quad y(0) = 1, y'(0) = 2, y''(0) = 1, y'''(0) = 0$$

(5 marks)

  c. Find the general solution given $y'' - 4y' + 4y = (x+1)e^{2x}$

(10 marks)

3.

  a. Use the method of undetermined coefficients to solve for y given

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = 2x^2 - 3x + 6 \qquad\qquad \text{(10 marks)}$$

  b. Use the method of the inverse D-operator to solve

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{3x} \qquad\qquad \text{(10 marks)}$$

4.

  a. A curve having a slope given by $\frac{dy}{dx} = \frac{2xy}{x^2-y^2}$, passes through point

  (2,1)

  Find its equation                                                         (10 marks)

  b. Let $y1 = cosx$ and $y2 = sinx$, find $\mathcal{W}$                     (5 marks)

  c. Solve $\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}}{x}$                         (5 marks)

5.

  a. Solve using Euler Cauchy method $x^3\frac{d^3y}{dx^3} + 5x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 8y = 0$

(5 marks)

a. Find the general solution of

    i.      $\dfrac{d^2y}{dx^2} - 8\dfrac{dy}{dx} + 25y = 5x^3e^{-x} - 7e^{-x}$                (5 marks)

    ii.    Find if $\dfrac{d^3y}{dx^3} - 6\dfrac{d^3y}{dx^3} + 11\dfrac{dy}{dx} - 6y = 0$ is linearly dependent

                                                                                      (5 marks)